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4 Modeling of FACTS-Devices in Optimal Power Flow Analysis

In recent years, energy, environment, deregulation of power utilities have delayed the construction of both generation facilities and new transmission lines. Better utilisation of existing power system capacities by installing new FACTS-devices has become imperative. FACTS-devices are able to change, in a fast and effective way, the network parameters in order to achieve a better system performance. FACTS-devices, such as phase shifter, shunt or series compensation and the most recent developed converter-based power electronic devices, make it possible to control circuit impedance, voltage angle and power flow for optimal operation of power systems, facilitate the development of competitive electric energy markets, and stipulate the unbundling the power generation from transmission and mandate open access to transmission services, etc.

However, in contrast to the practical applications of the STATCOM, SSSC and UPFC in power systems, very few publications have been focused on the mathematical modeling of these converter based FACTS-devices in optimal power flow analysis. This chapter covers

- Review of optimal power flow (OPF) solution techniques.
- Introduction of OPF solution by the nonlinear interior point methods.
- Mathematical modeling of FACTS-devices including STATCOM, SSSC, UPFC, IPFC, GUPFC, and VSC HVDC.
- The detailed models of the multi-converter FACTS-devices GUPFC, and VSC HVDC and their implementation into the nonlinear interior point OPF.
- Comparison of UPFC and VSC HVDC, and GUPFC and multiterminal VSC HVDC.
- Numerical examples for demonstration of FACTS controls

4.1 Optimal Power Flow Analysis

4.1.1 Brief History of Optimal Power Flow

The Optimal Power Flow (OPF) problem was initiated by the desire to minimize the operating cost of the supply of electric power when load is given [1][2]. In 1962 a generalized nonlinear programming formulation of the economic dispatch

problem including voltage and other operating constraints was proposed by Carpentier [3]. The Optimal Power Flow (OPF) problem was defined in early 1960's as an expansion of conventional economic dispatch to determine the optimal settings for control variables in a power network considering various operating and control constraints [4]. The OPF method proposed in [4] has been known as the reduced gradient method, which can be formulated by eliminating the dependent variables based on a solved load flow. Since the concept of the reduced gradient method for the solution of the OPF problem was proposed, continuous efforts in the developments of new OPF methods have been found. Several review papers were published [5]-[13]. Among the various OPF methods proposed, it has been recognized that the main techniques for solving the OPF problems are the gradient method [4], linear programming (LP) method [15][16], successive sparse quadratic programming (QP) method [18], successive non-sparse quadratic programming (QP) method [20], Newton's method [21] and Interior Point Methods [27]-[32]. Each method has its own advantages and disadvantages. These algorithms have been employed with varied success.

4.1.2 Comparison of Optimal Power Flow Techniques

It has been well recognised that the OPF problems are very complex mathematical programming problems. In the past, numerous papers on the numerical solution of the OPF problems have been published [7][10][11][13]. In this section, a review of several OPF methods is given.

4.1.2.1 Gradient Methods

The widely used gradient methods for the OPF problems include the reduced gradient method [4] and the generalised gradient method [14]. Gradient methods basically exhibit slow convergence characteristics near the optimal solution. In addition, the methods are difficult to solve in the presence of inequality constraints.

4.1.2.2 Linear Programming Methods

LP methods have been widely used in the OPF problems. The main strengths of LP based OPF methods are summarised as follows:

1. Efficient handling of inequalities and detection of infeasible solutions;
2. Dealing with local controls;
3. Incorporation of contingencies.

Noting the fact that it is quite common in the OPF problems, the nonlinear equalities and inequalities and objective function need to be handled. In this situation, all the nonlinear constraints and objective function should be linearized around the current operating point such that LP methods can be applied to solve the linear optimal problems. For a typical LP based OPF, the solution can be found through the iterations between load flow and linearized LP subproblem. The LP based OPF

methods have been shown to be effective for problems where the objectives are separable and convex. However, the LP based OPF methods may not be effective where the objective functions are non-separable, for instance in the minimization of transmission losses.

4.1.2.3 Quadratic Programming Methods

QP based OPF methods [17]-[20] are efficient for some OPF problems, especially for the minimization of power network losses. In [20], the non-sparse implementation of the QP based OPF was proposed while in [17][18][19], the sparse implementation of the QP based OPF algorithm for large-scale power systems was presented. In [17][18], the successive QP based OPF problems are solved through a sequence of linearly constrained subproblems using a quasi-Newton search direction. The QP formulation can always find a feasible solution by adding extra shunt compensation. In [19], the QP method, which is a direct solution method, solves a set of linear equations involving the Hessian matrix and the Jacobian matrix by converting the inequality constrained quadratic program (IQP) into the equality constrained quadratic program (EQP) with an initial guess at the correct active set. The computational speed of the QP method in [19] has been much improved in comparison to those in [17][18]. The QP methods in [17]-[19] are solved using MINOS developed by Stanford University.

4.1.2.4 Newton's Methods

The development of the OPF algorithm by Newton's method [21]-[24], is based on the success of the Newton's method for the power flow calculations. Sparse matrix techniques applied to the Newton power flow calculations are directly applicable to the Newton OPF calculations. The major idea is that the OPF problems are solved by the sequence of the linearized Newton equations where inequalities are being treated as equalities when they are binding. However, most critical aspect of the Newton's algorithm is that the active inequalities are not known prior to the solution and the efficient implementations of the Newton's method usually adopt the so-called trial iteration scheme where heuristic constraints enforcement/release is iteratively performed until acceptable convergence is achieved. In [22][25], alternative approaches using linear programming techniques have been proposed to identify the active set efficiently in the Newton's OPF.

In principle, the successive QP methods and Newton's method both using the second derivatives, which are considered as second order optimization method, are theoretically equivalent.

4.1.2.5 Interior Point Methods

Since Karmarkar published his paper on an interior point method for linear programming in 1984 [26], a great interest on the subject has arisen. Interior point methods have proven to be a promising alternative for the solution of power system optimization problems. In [27] and [28], a Security-Constrained Economic

Dispatch (SCED) is solved by sequential linear programming and the IP Dual-Affine Scaling (DAS). In [29], a modified IP DAS algorithm was proposed. In [30], an interior point method was proposed for linear and convex quadratic programming. It is used to solve power system optimization problems such as economic dispatch and reactive power planning. In [31]-[36], nonlinear primal-dual interior point methods for power system optimization problems were developed. The nonlinear primal-dual methods proposed can be used to solve the nonlinear power system OPF problems efficiently. The theory of nonlinear primal-dual interior point methods has been established based on three achievements: Fiacco & McCormick's barrier method for optimization with inequalities, Lagrange's method for optimization with equalities and Newton's method for solving nonlinear equations [37]. Experience with application of interior point methods to power system optimization problems has been quite positive.

4.1.3 Overview of OPF-Formulation

The OPF problem may be formulated as follows:

$$\text{Minimize: } f(\mathbf{x}, \mathbf{u}) \quad (4.1)$$

subject to:

$$\mathbf{g}(\mathbf{x}, \mathbf{u}) = 0 \quad (4.2)$$

$$\mathbf{h}_{\min} \leq \mathbf{h}(\mathbf{x}, \mathbf{u}) \leq \mathbf{h}_{\max} \quad (4.3)$$

where

$\mathbf{u}$ - the set of control variables

$\mathbf{x}$ - the set of dependent variables

$f(\mathbf{x}, \mathbf{u})$ - a scalar objective function

$\mathbf{g}(\mathbf{x}, \mathbf{u})$ - the power flow equations

$\mathbf{h}(\mathbf{x}, \mathbf{u})$ - the limits of the control variables and operating limits of power system components.

The objectives, controls and constraints of the OPF problems are summarized in Table 4.1. The limits of the inequalities in Table 4.1 can be classified into two categories: (a) physical limits of control variables; (b) operating limits of power system. In principle, physical limits on control variables can not be violated while operating limits representing security requirements can be violated or relaxed temporarily.

In addition to the steady state power flow constraints, for the OPF formulation, stability constraints, which are described by differential equations, may be considered and incorporated into the OPF. In recent years, stability constrained OPF problems have been proposed [38]-[42].

Table 4.1. Objectives, Constraints and Control Variables of the OPF Problems

Objectives	• Minimum cost of generation and transactions • Minimum transmission losses • Minimum shift of controls • Minimum number of controls shifted • Minimum number of controls rescheduled • Minimum cost of VAR investment
Equality constraints	• Power flow constraints • Other balance constraints
Inequality constraints	• Limits on all control variables • Branch flow limits (amps, MVA, MW, MVar) • Bus voltage variables • Transmission interface limits • Active/reactive power reserve limits
Controls	• Real and reactive power generation • Transformer taps • Generator voltage or reactive control settings • MW interchange transactions • HVDC link MW controls • FACTS voltage and power flow controls • Load shedding

4.2 Nonlinear Interior Point Optimal Power Flow Methods

4.2.1 Power Mismatch Equations

The power mismatch equations in rectangular coordinates at a bus are given by:

$$\Delta P_i = Pg_i - Pd_i - P_i \quad (4.4)$$

$$\Delta Q_i = Qg_i - Qd_i - Q_i \quad (4.5)$$

where Pg_i and Qg_i are real and reactive powers of generator at bus i , respectively; Pd_i and Qd_i the real and reactive load powers, respectively; P_i and Q_i the power injections at the node and are given by:

$$P_i = V_i \sum_{j=1}^N V_j (G_{ij} \cos \theta_{ij} + B_{ij} \sin \theta_{ij}) \quad (4.6)$$

$$Q_i = V_i \sum_{j=1}^N V_j (G_{ij} \sin \theta_{ij} - B_{ij} \cos \theta_{ij}) \quad (4.7)$$

where V_i and θ_i are the magnitude and angle of the voltage at bus i , respectively; $Y_{ij} = G_{ij} + jB_{ij}$ is the system admittance element while $\theta_{ij} = \theta_i - \theta_j$. N is the total number of system buses.

4.2.2 Transmission Line Limits

The transmission MVA limit may be represented by:

$$(P_{ij})^2 + (Q_{ij})^2 \leq (S_{ij}^{\max})^2 \quad (4.8)$$

where $S_{ij}^{\max}$ is the MVA limit of the transmission line ij . P_{ij} and Q_{ij} are given by:

$$P_{ij} = -V_i^2 G_{ij} + V_i V_j (G_{ij} \cos \theta_{ij} + B_{ij} \sin \theta_{ij}) \quad (4.9)$$

$$Q_{ij} = V_i^2 b_{ii} + V_i V_j (G_{ij} \sin \theta_{ij} - B_{ij} \cos \theta_{ij}) \quad (4.10)$$

where $b_{ii} = -B_{ij} + bc_{ij}/2$. bc_{ij} is the shunt admittance of transmission line ij .

4.2.3 Formulation of the Nonlinear Interior Point OPF

Mathematically, as an example the objective function of an OPF may minimize the total operating cost as follows:

$$\text{Minimize } f(x) = \sum_i^{N_g} (\alpha_i * Pg_i^2 + \beta_i * Pg_i + \gamma_i) \quad (4.11)$$

while being subject to the following constraints:

Nonlinear equality constraints:

$$\Delta P_i(x) = Pg_i - Pd_i - P_i(t, e, f) = 0 \quad (4.12)$$

$$\Delta Q_i(x) = Qg_i - Qd_i - Q_i(t, e, f) = 0 \quad (4.13)$$

Nonlinear inequality constraints

$$h_j^{\min} \leq h_j(x) \leq h_j^{\max} \quad (4.14)$$

where

$x = [Pg, Qg, t, \theta, V]^T$ is the vector of variables

$\alpha_i, \beta_i, \gamma_i$ coefficients of production cost functions of generator

$\Delta P(x)$ bus active power mismatch equations

$\Delta Q(x)$ bus reactive power mismatch equations

$h(x)$	functional inequality constraints including line flow and voltage magnitude constraints, simple inequality constraints of variables such as generator active power, generator reactive power, transformer tap ratio
Pg	the vector of active power generation
Qg	the vector of reactive power generation
t	the vector of transformer tap ratios
θ	the vector of bus voltage magnitude
V	the vector of bus voltage angle
Ng	the number of generators

By applying Fiacco and McCormick's barrier method, the OPF problem equations (4.11)-(4.14) can be transformed into the following equivalent OPF problem:

$$\text{Objective:} \quad \text{Min}\{f(x) - \mu \sum_{j=1}^M \ln(sl_j) - \mu \sum_{j=1}^M \ln(su_j)\} \quad (4.15)$$

subject to the following constraints:

$$\Delta P_i = 0 \quad (4.16)$$

$$\Delta Q_i = 0 \quad (4.17)$$

$$h_j - sl_j - h_j^{\min} = 0 \quad (4.18)$$

$$h_j + su_j - h_j^{\max} = 0 \quad (4.19)$$

where $sl > 0$ and $su > 0$.

Thus the Lagrangian function for equalities optimisation of equations (4.15)-(4.19) is given by:

$$\begin{aligned} L = & f(x) - \mu \sum_{j=1}^M \ln(sl_j) - \mu \sum_{j=1}^M \ln(su_j) - \sum_{i=1}^N \lambda p_i \Delta P_i - \sum_{i=1}^N \lambda q_i \Delta Q_i \\ & - \sum_{j=1}^M \pi l_j (h_j - sl_j - h_j^{\min}) - \sum_{j=1}^M \pi u_j (h_j + su_j - h_j^{\max}) \end{aligned} \quad (4.20)$$

where λp_i , λq_i , πl_j , πu_j are Langrage multipliers for the constraints of equations (4.16)-(4.19), respectively. N represents the number of buses and M the number of inequality constraints. Note that $\mu > 0$. The Karush-Kuhn-Tucker (KKT) first order conditions for the Lagrangian function shown in equation (4.20) are as follows:

$$\nabla_x L_\mu = \nabla f(x) - \nabla \Delta P^T \lambda p - \nabla \Delta Q^T \lambda q - \nabla h^T \pi l - \nabla h^T \pi u = 0 \quad (4.21)$$

$$\nabla_{\lambda p} L_\mu = -\Delta P = 0 \quad (4.22)$$

$$\nabla_{\lambda_q} L_\mu = -\Delta Q = 0 \quad (4.23)$$

$$\nabla_{\pi_l} L_\mu = -(h - sl - h^{\min}) = 0 \quad (4.24)$$

$$\nabla_{\pi_u} L_\mu = -(h + su - h^{\max}) = 0 \quad (4.25)$$

$$\nabla_{sl} L_\mu = \mu - S\Pi l = 0 \quad (4.26)$$

$$\nabla_{su} L_\mu = \mu + Su\Pi u = 0 \quad (4.27)$$

where $Sl = \text{diag}(sl_j)$, $Su = \text{diag}(su_j)$, $\Pi l = \text{diag}(\pi_l_j)$, $\Pi u = \text{diag}(\pi_u_j)$.

As suggested in [31], the above equations can be decomposed into the following three sets of equations:

$$\begin{bmatrix} -\Pi l^{-1}Sl & 0 & -\nabla h & 0 \\ 0 & \Pi u^{-1}Su & -\nabla h & 0 \\ -\nabla h^T & -\nabla h^T & H & -J^T \\ 0 & 0 & -J & 0 \end{bmatrix} \begin{bmatrix} \Delta \pi_l \\ \Delta \pi_u \\ \Delta x \\ \Delta \lambda \end{bmatrix} = \begin{bmatrix} -\nabla_{\pi_l} L_\mu - \Pi l^{-1} \nabla_{Sl} L_\mu \\ -\nabla_{\pi_u} L_\mu - \Pi u^{-1} \nabla_{Su} L_\mu \\ -\nabla_x L_\mu \\ -\nabla_{\lambda} L_\mu \end{bmatrix} \quad (4.28)$$

$$\Delta sl = \Pi l^{-1} (\nabla_{sl} L_\mu - Sl \Delta \pi_l) \quad (4.29)$$

$$\Delta su = \Pi u^{-1} (-\nabla_{su} L_\mu - Su \Delta \pi_u) \quad (4.30)$$

where $H(x, \lambda, \pi_l, \pi_u) = \nabla^2 f(x) - \sum \lambda \nabla^2 g(x) - \sum (\pi_l + \pi_u) \nabla^2 h(x)$,

$$J(x) = \begin{bmatrix} \frac{\partial \Delta P(x)}{\partial x} & \frac{\partial \Delta Q(x)}{\partial x} \end{bmatrix}, \quad g(x) = \begin{bmatrix} \Delta P(x) \\ \Delta Q(x) \end{bmatrix}, \quad \text{and } \lambda = \begin{bmatrix} \lambda_p \\ \lambda_q \end{bmatrix}.$$

The elements corresponding to the slack variables sl and su have been eliminated from equation (4.28) using analytical Gaussian elimination. By solving equation (4.28), $\Delta \pi_l$, $\Delta \pi_u$, Δx , $\Delta \lambda$ can be obtained, then by solving equations (4.29) and (4.30), respectively, Δsl , Δsu can be obtained. With $\Delta \pi_l$, $\Delta \pi_u$, Δx , $\Delta \lambda$, Δsl , Δsu known, the OPF solution can be updated using the following equations:

$$sl^{(k+1)} = sl^{(k)} + \sigma \alpha_p \Delta sl \quad (4.31)$$

$$su^{(k+1)} = su^{(k)} + \sigma \alpha_p \Delta su \quad (4.32)$$

$$x^{(k+1)} = x^{(k)} + \sigma \alpha_p \Delta x \quad (4.33)$$

$$\pi_l^{(k+1)} = \pi_l^{(k)} + \sigma \alpha_d \Delta \pi_l \quad (4.34)$$

$$\pi_u^{(k+1)} = \pi_u^{(k)} + \sigma \alpha_d \Delta \pi_u \quad (4.35)$$

$$\pi u^{(k+1)} = \pi u^{(k)} + \sigma \alpha_d \Delta \pi u \quad (4.36)$$

$$\lambda^{(k+1)} = \lambda^{(k)} + \sigma \alpha_d \Delta \lambda \quad (4.37)$$

where k is the iteration count, parameter $\sigma \in [0.995 - 0.99995]$ and α_p and α_d are the primal and dual step-length parameters, respectively. The step-lengths are determined as follows:

$$\alpha_p = \min \left[\min \left(\frac{sl}{-\Delta sl} \right), \min \left(\frac{su}{-\Delta su} \right), 1.00 \right] \quad (4.38)$$

$$\alpha_d = \min \left[\min \left(\frac{\pi l}{-\Delta \pi l} \right), \min \left(\frac{\pi u}{-\Delta \pi u} \right), 1.00 \right] \quad (4.39)$$

for those $sl < 0$, $\Delta su < 0$, $\Delta \pi l < 0$ and $\Delta \pi u > 0$.

The barrier parameter μ can be evaluated by:

$$\mu = \frac{\beta \times Cgap}{2 \times M} \quad (4.40)$$

where $\beta \in [0.01-0.2]$ and $Cgap$ is the complementary gap for the nonlinear interior point OPF and can be determined using:

$$Cgap = \sum_{j=1}^M (sl_j \pi l_j - su_j \pi u_j) \quad (4.41)$$

4.2.4 Implementation of the Nonlinear Interior Point OPF

Equations (4.28)-(4.30) are the basic formulation of the nonlinear interior point OPF that has been well reported in [31][51]. Equation (4.28) is the reduced equation with respect to the original OPF problem. However, equation (4.28) can be further reduced by eliminating all the dual variables of the inequalities, generator output variables and transformer tap ratios. The elimination will result in new fill-in elements. For example, eliminating a transformer tap ratio will result in sixteen new elements in the reduced Newton equation. Such a significant reduction means that the reduced Newton equation only involves the state variables of θ_i , V_i , λp_i , λq_i . The details will be discussed in the next section.

4.2.4.1 Eliminating Dual Variables πl , πu of the Inequalities

In order to obtain the final reduced Newton equation consisting of only the variables θ , V , λp , λq , the following Gaussian elimination steps can be applied.

The dimension of the Newton equation (4.28) can be reduced using analytical Gaussian elimination techniques. Basically, the dual variables πl , πu in equation (4.28) can be eliminated to obtain:

$$H' \Delta x - J_P^T \Delta \lambda p - J_Q^T \Delta \lambda q = -\nabla_x \dot{L}_\mu \quad (4.42)$$

$$-J_P \Delta \lambda p = -\nabla_{\lambda p} L_\mu \quad (4.43)$$

$$-J_Q \Delta \lambda q = -\nabla_{\lambda q} L_\mu \quad (4.44)$$

$$H' \Delta x - J_P^T \Delta \lambda p - J_Q^T \Delta \lambda q = -\nabla_x \dot{L}_\mu \quad (4.45)$$

where:

$$H' = H + \nabla h^T \nabla h (\Pi l S l^{-1} - \Pi u S u^{-1}) \quad (4.46)$$

$$\begin{aligned} \nabla_x \dot{L}_\mu = & \nabla_x L_\mu - \nabla h^T S l^{-1} (\nabla_{sl} L_\mu + \Pi l \nabla_{\pi l} L_\mu) \\ & + \nabla h^T S u^{-1} (\nabla_{su} L_\mu + \Pi u \nabla_{\pi u} L_\mu) \end{aligned} \quad (4.47)$$

$$J_P = \frac{\partial \Delta P(x)}{\partial x} \quad (4.48)$$

$$J_Q = \frac{\partial \Delta Q(x)}{\partial x} \quad (4.49)$$

The equations (4.42)-(4.45) can be written as the following compact form:

$$\begin{bmatrix} H' & -J_P^T & -J_Q^T \\ -J_P & 0 & 0 \\ -J_Q & 0 & 0 \end{bmatrix} \cdot \begin{bmatrix} \Delta x \\ \Delta \lambda p \\ \Delta \lambda q \end{bmatrix} = \begin{bmatrix} -\nabla_x \dot{L}_\mu \\ -\nabla_{\lambda p} L_\mu \\ -\nabla_{\lambda q} L_\mu \end{bmatrix} \quad (4.50)$$

By solving equation (4.50), Δx can be obtained, then the dual variables $\Delta \pi l$ and $\Delta \pi u$ can be found by solving the following equations:

$$\Delta \pi l = -\Pi l S l^{-1} (\nabla h \Delta x - \nabla_{\pi l} L_\mu) + S l^{-1} \nabla_{sl} L_\mu \quad (4.51)$$

$$\Delta \pi u = \Pi u S u^{-1} (\nabla h \Delta x - \nabla_{\pi u} L_\mu) - S u^{-1} \nabla_{su} L_\mu \quad (4.52)$$

Up to now, equation (4.28) has been reduced to three lower dimension equations (4.50), (4.51) and (4.52). In (4.50), all inequalities have been eliminated, while equations (4.51) and (4.52) are relatively simple to solve.

4.2.4.2 Eliminating Generator Variables P_g and Q_g

In equation (4.50), generator variables P_g , Q_g can be further eliminated. The equation (4.50) may be written in the following form, in which only the relevant major diagonal block of bus i is displayed:

$$\begin{bmatrix} H' P_{gi} P_{gi} & 0 & 0 & 0 & -1 & 0 \\ 0 & H' Q_{gi} Q_{gi} & 0 & 0 & 0 & -1 \\ 0 & 0 & H' \theta_i \theta_i & H' \theta_i V_i & -J_{p_i, \theta_i} & -J_{q_i, \theta_i} \\ 0 & 0 & H' V_i \theta_i & H' V_i V_i & -J_{p_i, V_i} & -J_{q_i, V_i} \\ -1 & 0 & -J_{p_i, \theta_i} & -J_{p_i, V_i} & 0 & 0 \\ 0 & -1 & -J_{q_i, \theta_i} & -J_{q_i, V_i} & 0 & 0 \end{bmatrix} \times \begin{bmatrix} \Delta P_{gi} \\ \Delta Q_{gi} \\ \Delta \theta_i \\ \Delta V_i \\ \Delta \lambda_{p_i} \\ \Delta \lambda_{q_i} \end{bmatrix} = \begin{bmatrix} -\nabla_{P_{gi}} \dot{L}_\mu \\ -\nabla_{Q_{gi}} \dot{L}_\mu \\ -\nabla_{\theta_i} \dot{L}_\mu \\ -\nabla_{V_i} \dot{L}_\mu \\ -\nabla_{\lambda_{p_i}} \dot{L}_\mu \\ -\nabla_{\lambda_{q_i}} \dot{L}_\mu \end{bmatrix} \quad (4.53)$$

Eliminating ΔP_{gi} and ΔQ_{gi} from the above equation, we have:

$$\begin{bmatrix} H' \theta_i \theta_i & H' \theta_i V_i & -J_{p_i, \theta_i} & -J_{q_i, \theta_i} \\ H' V_i \theta_i & H' V_i V_i & -J_{p_i, V_i} & -J_{q_i, V_i} \\ -J_{p_i, \theta_i} & -J_{p_i, V_i} & -J \lambda_{p_i} & 0 \\ -J_{q_i, \theta_i} & -J_{q_i, V_i} & 0 & -J \lambda_{q_i} \end{bmatrix} \begin{bmatrix} \Delta \theta_i \\ \Delta V_i \\ \Delta \lambda_{p_i} \\ \Delta \lambda_{q_i} \end{bmatrix} = \begin{bmatrix} -\nabla_{\theta_i} \dot{L}_\mu \\ -\nabla_{V_i} \dot{L}_\mu \\ -\nabla_{\lambda_{p_i}} \dot{L}_\mu \\ -\nabla_{\lambda_{q_i}} \dot{L}_\mu \end{bmatrix} \quad (4.54)$$

where:

$$\begin{aligned} J \lambda_{p_i} &= (H' P_{gi} P_{gi})^{-1} \\ J \lambda_{q_i} &= (H' Q_{gi} Q_{gi})^{-1} \\ \nabla_{\lambda_{p_i}} \dot{L}_\mu &= \nabla_{\lambda_{p_i}} L_\mu + (H' P_{gi} P_{gi})^{-1} \nabla_{P_{gi}} \dot{L}_\mu \\ \nabla_{\lambda_{q_i}} \dot{L}_\mu &= \nabla_{\lambda_{q_i}} L_\mu + (H' Q_{gi} Q_{gi})^{-1} \nabla_{Q_{gi}} \dot{L}_\mu \end{aligned}$$

By solving equation (4.54), $\Delta \lambda_{p_i}$ and $\Delta \lambda_{q_i}$ can be obtained, then ΔP_{gi} and ΔQ_{gi} can be found by the following equations:

$$\Delta P_{gi} = (H' P_{gi} P_{gi})^{-1} (\Delta \lambda_{p_i} - \nabla_{P_{gi}} \dot{L}_\mu) \quad (4.55)$$

$$\Delta Q_{gi} = (H' Q_{gi} Q_{gi})^{-1} (\Delta \lambda_{q_i} - \nabla_{Q_{gi}} \dot{L}_\mu) \quad (4.56)$$

Similarly, the elements corresponding to the transformer tap ratio can be eliminated using the same principle resulting in a reduced Newton equation consisting of only the variables θ , V , λ_p and λ_q . Since the reduced Newton equation has the similar structure as that of (4.54), in the next discussions the final reduced Newton equation (without considering FACTS) is still referred to (4.54). However, it should be pointed out that the elimination of the transformer tap ratio will affect the sparsity of the matrix because new fill in elements are resulting.

4.2.5 Solution Procedure for the Nonlinear Interior Point OPF

The solution of the nonlinear interior point OPF may be summarised as follows:

Step 0: Formulation of equation (4.28)

Step 1: Forward substitution

- (a) Eliminating the dual variables π_l , π_u of the inequalities from equation (4.28), obtain equation (4.50);
- (b) Eliminating generator variables P_g , Q_g from equation (4.50), obtain equation (4.54);
- (c) Eliminating the transformer tap ratio t from equation (4.54), obtain highly reduced Newton matrix equation.

Step 2: Solution of the final highly reduced Newton equation by sparse matrix techniques

- (a) The highly reduced system matrix has a dimension of $4N$ where N is the total number of buses.
- (b) Having been grouped into 4 by 4 blocks, solution to the final matrix is produced by sparse matrix techniques.

Step 3: Back substitution

- (a) First substitution for transformer tap ratio. After solving the final matrix equation, $\Delta\theta$, ΔV , $\Delta\lambda_p$ and $\Delta\lambda_q$ are known, then Δt can be found by back substitution.
- (b) Second substitution for generator output variables: ΔP_g and ΔQ_g can be found from equations (4.55) and (4.56);
- (c) Third substitution for all the dual variables of the inequalities: The dual variables π_l , π_u of the inequalities can be found by equation (4.51) and equation (4.52);
- (d) Fourth substitution for all slack variables: All slack variables can be found by equations (4.29) and (4.30).

4.3 Modeling of FACTS in OPF Analysis

Very few publications have been focused on the mathematical modeling of FACTS-devices in optimal power flow analysis. In [44][45], a UPFC model has been proposed, and the model has been implemented in a Successive QP. In [46], mathematical models for TCSC, IPC and UPFC have been established, and the OPF problem with these FACTS-devices is solved by Newton's method. In [47], a versatile model for UPFC in OPF analysis has been proposed and the model has been implemented into the nonlinear interior point methods. In this model, explicit controls such local voltage and power flow controls can be explicitly represented. Furthermore, in this model, global controls of UPFC can be achieved without explicit controls. In [48], the modeling techniques in [47] have been extended to

general mathematical models for the converter based FACTS-devices such as STATCOM, SSSC, and UPFC suitable for optimal power flow analysis. Applying the techniques in chapter 3, the Thyristor controlled FACTS-devices such as SVC and TCSC can be modeled in OPF analysis. The detailed models of STATCOM, SSSC, UPFC are referred to [47][48]. In the next sections, novel models for IPFC, GUPFC, multi-terminal VSC HVDC will be discussed, the modeling techniques of which are applicable to STATCOM, SSSC and UPFC.

4.3.1 IPFC and GUPFC in Optimal Voltage and Power Flow Control

An innovative approach to utilization of FACTS-devices providing a multifunctional power flow management device was proposed in [43]. There are several possibilities of operating configurations by combining two or more converter blocks with flexibility. Among them, there are two novel operating configurations, namely the Interline Power Flow Controller (IPFC) and the Generalized Unified Power Flow Controller (GUPFC) [43][49][50], which are significantly extended to control power flows of multi-lines or a sub-network rather than controlling the power flow of a single line by a UPFC or SSSC.

In contrast to the practical applications of the GUPFC in power systems, very few publications have been focused on the mathematical modeling of this new FACTS-device in power system analysis. A fundamental frequency model of the GUPFC consisting of one shunt converter and two series converters for EMTP study was proposed quite recently in [50]. The modeling of IPFC and GUPFC in power flow and optimal power flow (OPF) analysis has been reported [51][52]. In the next, novel model for GUPFC will be proposed, which are very convenient to consider various control constraints and control modes. The model for IPFC can be very easily derived once the model for GUPFC has been established.

4.3.2 Operating and Control Constraints of GUPFC

As discussed in chapter 3, the GUPFC combining three or more converters working together extends the concepts of voltage and power flow control beyond what is achievable with the known two-converter UPFC. The simplest GUPFC consists of three converters, one connected in shunt and the other two in series with two transmission lines in a substation. It can control total five power system quantities such as a bus voltage and independent active and reactive power flows of two lines. The equivalent circuit of such a GUPFC, which is shown in Fig. 4.1, is used to show the basic operation principle for the sake of simplicity. However, the mathematical derivation is applicable to a GUPFC with an arbitrary number of series converters.

In the steady state operation, the main objective of the GUPFC is to control voltage and power flow. Real power can be exchanged among these shunt and series converters via the common DC link. The sum of the real power exchange should be zero if we neglect the losses of the converter circuits.

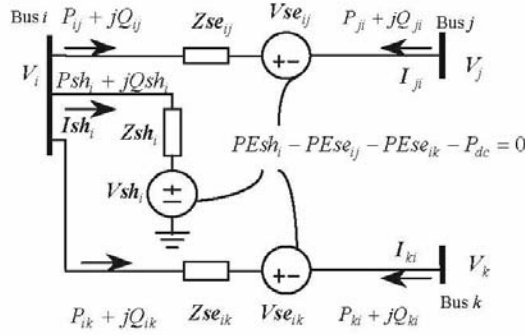


Fig. 4.1. The equivalent circuit of the GUPFC

For the GUPFC shown in Fig. 4.1, it has total 5 degrees of control freedom, that means it can control five power system quantities such as one bus voltage, and 4 active and reactive power flows of two lines. It can be seen that with more series converters included within the GUPFC, more degrees of control freedom can be introduced and hence more control objectives can be achieved.

In Fig. 4.1 Zsh_i and Zse_{in} are the shunt and series transformer impedances, respectively. $Vsh_i = Vsh_i \angle \theta sh_i$ and $Vse_{in} = Vse_{in} \angle \theta se_{in}$ ($n = j, k$) are the controllable injected shunt and series voltage sources. $PEsh_i$ and $PEse_{in}$ are the power exchange of the shunt converter and series converter, respectively, via the common DC link.

4.3.2.1 Power Flow Constraints of GUPFC

The power flow constraints of the GUPFC are summarized as follows:

Shunt power flows:

$$Psh_i = V_i^2 gsh_i - V_i Vsh_i (gsh_i \cos(\theta_i - \theta sh_i) + bsh_i \sin(\theta_i - \theta sh_i)) \quad (4.57)$$

$$Qsh_i = -V_i^2 bsh_i - V_i Vsh_i (gsh_i \sin(\theta_i - \theta sh_i) - bsh_i \cos(\theta_i - \theta sh_i)) \quad (4.58)$$

Series power flows:

$$P_{in} = V_i^2 g_{in} - V_i V_n (g_{in} \cos \theta_{in} + b_{in} \sin \theta_{in}) - V_i Vse_{in} (g_{in} \cos(\theta_i - \theta se_{in}) + b_{in} \sin(\theta_i - \theta se_{in})) \quad (4.59)$$

$$Q_{in} = -V_i^2 b_{in} - V_i V_n (g_{in} \sin \theta_{in} - b_{in} \cos \theta_{in}) - V_i Vse_{in} (g_{in} \sin(\theta_i - \theta se_{in}) - b_{in} \cos(\theta_i - \theta se_{in})) \quad (4.60)$$

$$P_{ni} = V_n^2 g_{in} - V_i V_n (g_{in} \cos(\theta_n - \theta_i) + b_{in} \sin(\theta_n - \theta_i)) \\ + V_n V_{se_{in}} (g_{in} \cos(\theta_n - \theta_{se_{in}}) + b_{in} \sin(\theta_n - \theta_{se_{in}})) \quad (4.61)$$

$$Q_{ni} = -V_n^2 b_{nn} - V_i V_n (g_{in} \sin(\theta_n - \theta_i) - b_{in} \cos(\theta_n - \theta_i)) \\ + V_n V_{se_{in}} (g_{in} \sin(\theta_n - \theta_{se_{in}}) - b_{in} \cos(\theta_n - \theta_{se_{in}})) \quad (4.62)$$

where $g_{in} = \text{Re}(1/\mathbf{Z}_{se_{in}})$, $b_{in} = \text{Im}(1/\mathbf{Z}_{se_{in}})$. P_{in} , Q_{in} ($n=j, k$) are the active and reactive power flows of two GUPFC series branches leaving bus i while P_{ni} , Q_{ni} ($n=j, k$) are the active and reactive power flows of the GUPFC series branch $n-i$ leaving bus n ($n=j, k$), respectively. Since two transmission lines are series connected with the FACTS branches $i-j$, $i-k$ via the GUPFC buses j and k , respectively, P_{ni} , Q_{ni} ($n=j, k$) are equal to the active and reactive power flows at the sending-end of the transmission lines, respectively.

The operating constraint representing the active power exchange among converters via the common DC link is:

$$PEx = Pesh_i - \sum PEse_{in} - P_{dc} = 0 \quad (4.63)$$

where $n=j, k$. P_{dc} is the power loss of the DC circuit of the GUPFC. $Pesh_i$ and $PEse_{in}$ satisfy the following equalities:

$$Pesh_i - \text{Re}(\mathbf{V}sh_i \mathbf{I}sh_i^*) = 0 \quad (4.64)$$

$$PEse_{in} - \text{Re}(\mathbf{V}se_{in} \mathbf{I}_{ni}^*) = 0 \quad (4.65)$$

4.3.2.2 Operating Control Equalities of GUPFC

The GUPFC shown in Fig. 4.1 can control both active and reactive power flows of the two transmission lines. The active and reactive power flow control constraints of the GUPFC are given by (3.50) and (3.51)

The GUPFC has additional capability to control the voltage magnitude of bus i :

$$V_i - V_i^{Spec} = 0 \Leftrightarrow V_i^{Spec} - \varepsilon \leq V_i \leq V_i^{Spec} + \varepsilon \quad (4.66)$$

where V_i is the voltage magnitude at bus i . V_i^{Spec} is the specified bus voltage control reference at bus i . In the point of view of the implementation, the inequality is preferred since incorporation of the simple variable inequality is very easy.

4.3.2.3 Operating Inequalities of GUPFC

For the operation of the GUPFC, the injected voltage sources should be within their operating ratings while the currents through the converters should be within the current ratings:

Shunt converter

$$\theta sh_i^{\min} \leq \theta sh_i \leq \theta sh_i^{\max} \quad (4.67)$$

$$Vsh_i^{\min} \leq Vsh_i \leq Vsh_i^{\max} \quad (4.68)$$

$$-PEsh_i^{\max} \leq PEsh_i \leq PEsh_i^{\max} \quad (4.69)$$

$$Ish_i \leq Ish_i^{\max} \quad (4.70)$$

Series converter

$$\theta se_{in}^{\min} \leq \theta se_{in} \leq \theta se_{in}^{\max} \quad (4.71)$$

$$Vse_{in}^{\min} \leq Vse_{in} \leq Vse_{in}^{\max} \quad (4.72)$$

$$-PEse_{in}^{\max} \leq PEse_{in} \leq PEse_{in}^{\max} \quad (4.73)$$

$$I_{ni} \leq I_{ni}^{\max} \quad (4.74)$$

where $n = j, k$. $PEsh_i^{\max}$ is the maximum limit of the power exchange of the shunt converter with the DC link. $Ish_i^{\max}$ is the current rating. $PEse_{in}^{\max}$ is the maximum limit of the power exchange of the series converter with the DC link ($n = j, k$). $I_{ni}^{\max}$ is the current rating of the series converter.

4.3.3 Incorporation of GUPFC into Nonlinear Interior Point OPF

4.3.3.1 Constraints of GUPFC

The GUPFC consists of the power flow constraints (4.57)-(4.62), the internal power exchange balance constraint (4.63), the operating inequality constraints (4.67)-(4.74), and the power flow control constraints and voltage control constraint (4.66). In the formulation of the nonlinear interior point OPF algorithm, the power flow constraints (4.57)-(4.62) can be directly incorporated into the power mismatch equations at bus i, j and k . By introducing slack variables and barrier parameter, the inequalities (4.67)-(4.74) can be converted into equalities. Then all the transformed equalities of the GUPFC can be incorporated into the Lagrangian function of the OPF problem.

4.3.3.2 Variables of GUPFC

The state variables of the GUPFC are $\theta sh_i, Vsh_i, PEsh_i, \theta se_{in}, Vse_{in}, PEse_{in}$. With incorporation of $PEsh_i, PEse_{in}$ into the state variables of the GUPFC, the formulation of the Newton OPF equation and the implementation of multi-control

functional model become simple and straightforward. In addition to the state variables, dual variables should be introduced for all the equalities and inequalities while slack variables should be introduced for all the inequalities. In the implementation, the angle constraints (4.67) and (4.71) are optional since they are usually allowed to move around 360° . For the simplicity of presentation, the angle constraints (4.67) and (4.71) are not discussed here.

The dual variables of the GUPFC inequality constraints are defined as follows:

$$\pi Vsh_i : \quad Vsh_i - Vsh_i^{\min} - SlVsh_i = 0 \quad (4.75)$$

$$\mu Vsh_i : \quad Vsh_i - Vsh_i^{\max} + SuVsh_i = 0 \quad (4.76)$$

$$\pi PEsh_i : \quad PEsh_i - PEsh_i^{\min} - SlPEsh_i = 0 \quad (4.77)$$

$$\mu PEsh_i : \quad PEsh_i - PEsh_i^{\max} + SuPEsh_i = 0 \quad (4.78)$$

$$\mu Ish_i : \quad Ish_i - Ish_i^{\max} + SuIsh_i = 0 \quad (4.79)$$

$$\pi Vse_{in} : \quad Vse_{in} - Vse_{in}^{\min} - SlVse_{in} = 0 \quad (4.80)$$

$$\mu Vse_{in} : \quad Vse_{in} - Vse_{in}^{\max} + SuVse_{in} = 0 \quad (4.81)$$

$$\pi PEse_{in} : \quad PEse_{in} - PEse_{in}^{\min} - SlPEse_{in} = 0 \quad (4.82)$$

$$\mu PEse_{in} : \quad PEse_{in} - PEse_{in}^{\max} + SuPEse_{in} = 0 \quad (4.83)$$

$$\mu Ise_{ni} : \quad Ise_{ni} - Ise_{ni}^{\max} + SuIse_{ni} = 0 \quad (4.84)$$

In the above equations, all the variables that start with ‘S’ are slack variables and they are positive values while all the variables that start with ‘ π ’ are dual variables.

The dual variables of the equalities of the GUPFC are defined as follows:

$$\lambda PEx : \quad PEx = PEsh_i + \sum PEse_{in} = 0 \quad (4.85)$$

$$\lambda PEsh_i : \quad PEsh_i - \text{Re}(\mathbf{Vsh}_i \mathbf{Ish}_i^*) = 0 \quad (4.86)$$

$$\lambda PEse_{in} : \quad PEse_{in} - \text{Re}(\mathbf{Vse}_{in} \mathbf{I}_{in}^*) = 0 \quad (4.87)$$

$$\lambda P_{ni} : \quad \Delta P_{ni} = P_{ni} - P_{ni}^{Spec} = 0 \quad (4.88)$$

$$\lambda Q_{ni} : \quad \Delta Q_{ni} = Q_{ni} - Q_{ni}^{Spec} = 0 \quad (4.89)$$

$$\lambda p_m : \quad \Delta P_m = 0 \quad (m = i, j, k) \quad (4.90)$$

$$\lambda q_m : \quad \Delta Q_m = 0 \quad (m = i, j, k) \quad (4.91)$$

where ΔP_m and ΔQ_m are power mismatch equations at bus m .

4.3.3.3 Augmented Lagrangian Function of GUPFC in Nonlinear Interior OPF

The augmented Lagrangian function of the equalities (4.75) -(4.84) is as follows:

$$\pi * \text{equality} - \mu \ln(S) \quad (4.92)$$

The augmented Lagrangian function of the equalities (4.85)-(4.91) is defined as follows:

$$\begin{aligned} & -\lambda PEx(PEsh_i - \sum_{n=j,k} PEse_{in}) - \lambda PEsh_i(PEsh_i - \text{Re}(Vsh_i Ish_i^*)) \\ & -\lambda PEse_{in}(PEse_{in} - \text{Re}(Vse_{in} Ise_{in}^*)) \\ & -\lambda P_{ni}(P_{ni} - P_{ni}^{Spec}) - \lambda Q_{ni}(Q_{ni} - Q_{ni}^{Spec}) \\ & -\lambda p_m \Delta P_m - \lambda q_m \Delta Q_m \end{aligned} \quad (4.93)$$

($n = j, k; m = i, j, k$)

4.3.3.4 Newton Equation of Nonlinear Interior OPF with GUPFC

With the incorporation of the augmented Lagrangian functions above into the OPF problem in section 4.2, a reduced Newton equation can be derived:

$$\left[\begin{array}{c|c} \mathbf{A} & \mathbf{C} \\ \hline \mathbf{C}^T & \mathbf{B} \end{array} \right] \left[\begin{array}{c} \frac{\Delta \mathbf{x}^{gupfc}}{\Delta \mathbf{x}^{sys}} \end{array} \right] = \left[\begin{array}{c} \mathbf{a} \\ \mathbf{b} \end{array} \right] \quad (4.94)$$

where

$\Delta \mathbf{X}^{gupfc} = [\Delta \mathbf{X}_{ik}^{gupfc}, \Delta \mathbf{X}_{ij}^{gupfc}, \Delta \mathbf{X}_i^{gupfc}]^T$ - the incremental vector of the GUPFC variables, and

$\Delta \mathbf{X}_{in}^{gupfc} = [\Delta \pi Ise_{ni}, \Delta \theta se_{in}, \Delta Vse_{in}, \Delta PEse_{in}, \lambda PEse_{in}, \Delta \lambda Pse_{ni}, \Delta \lambda Qse_{ni}]^T$ - the incremental vector of the variables of the GUPFC series branch in .

$\Delta \mathbf{X}_i^{gupfc} = [\Delta \pi Ish_i, \Delta \theta sh_i, \Delta Vsh_i, \Delta PEsh_i, \Delta \lambda PEsh_i, \Delta \lambda PEx]^T$ - the incremental vector of the variables of the GUPFC shunt branch i .

$\Delta \mathbf{X}^{sys} = [\Delta \mathbf{X}_i^{sys}, \Delta \mathbf{X}_j^{sys}, \Delta \mathbf{X}_k^{sys}]^T$ - the incremental vector of the variables of the system buses.

$\mathbf{a} = [\mathbf{a}_{ij}, \mathbf{a}_{ik}, \mathbf{a}_i]^T$ - the right hand vector of the GUPFC.

$\Delta \mathbf{X}_m^{sys} = [\Delta \theta_m, \Delta V_m, \Delta \lambda p_m, \Delta \lambda q_m]^T$ ($m = i, j, k$) - the incremental vector of the variables of system bus m .

In (4.94), all the slack and dual variables of the simple variable inequalities have been eliminated from the formulation. $\mathbf{B}$ and $\mathbf{b}$ are the system matrix and right hand vector, which have similar structure to the system matrix and right hand of (4.54), respectively except that in calculating the former, the contributions from the GUPFC should be considered. $\mathbf{a}_{in}$ and $\mathbf{a}_i$ are given by

$$\mathbf{a}_{in} = \begin{bmatrix} -\nabla_{\pi Ise_{in}} L_{\mu} - (\pi Ise_{in})^{-1} \nabla_{SuIse_{in}} L_{\mu} \\ -\nabla_{\theta se_{in}} L_{\mu} \\ -\nabla_{Vse_{in}} L_{\mu} + \mu(1/SIVse_{in} - 1/SuVse_{in}) \\ -\nabla_{PEse_{in}} L_{\mu} + \mu(1/SIPEse_{in} - 1/SuPEse_{in}) \\ -\nabla_{\lambda PEse_{in}} L_{\mu} \\ -\nabla_{\lambda Pse_{in}} L_{\mu} \\ -\nabla_{\lambda Qse_{in}} L_{\mu} \end{bmatrix} \quad (n = j, k) \quad (4.95)$$

$$\mathbf{a}_i = \begin{bmatrix} -\nabla_{\pi Ish_i} L_{\mu} - (\pi Ish_i)^{-1} \nabla_{SuIsh_i} L_{\mu} \\ -\nabla_{\theta sh_i} L_{\mu} \\ -\nabla_{Vsh_i} L_{\mu} + \mu(1/SIVsh_i - 1/SuVsh_i) \\ -\nabla_{PEsh_i} L_{\mu} + \mu(1/SIPEsh_i - 1/SuPEsh_i) \\ -\nabla_{\lambda PEsh_i} L_{\mu} \\ -\nabla_{\lambda PEx} L_{\mu} \end{bmatrix} \quad (4.96)$$

In (4.54), $\mathbf{A}$ and $\mathbf{C}$ are given by:

$$\mathbf{A} = \left[\frac{\partial^2 L}{\partial \mathbf{X}^{gupfc} \partial \mathbf{X}^{gupfc}} \right] + \mathbf{D} \quad (4.97)$$

$$\mathbf{C} = \left[\frac{\partial^2 L}{\partial \mathbf{X}^{gupfc} \partial \mathbf{X}^{sys}} \right] \quad (4.98)$$

where $\mathbf{D}$ is given by:

$$\mathbf{D} = \text{Diag}[\mathbf{d}_{ij}, \mathbf{d}_{ik}, \mathbf{d}_i] \quad (4.99)$$

$$\mathbf{d}_{in} = \text{Diag} \begin{bmatrix} 0, 0, (\pi Vse_{in}/SIVse_{in} - \pi Vse_{in}/SuVse_{in}) \\ (\pi PEse_{in}/SIPEse_{in} - \pi PEse_{in}/SuPEse_{in}), 0, 0, 0 \end{bmatrix} \quad (4.100)$$

($n = j, k$)

$$\mathbf{d}_i = \text{Diag} \begin{bmatrix} 0, 0, (\pi V_{se_i} / SIV_{se_i} - \pi u V_{se_i} / SuV_{se_i}) \\ (\pi PE_{se_i} / SIPE_{se_i} - \pi u PE_{se_i} / SuPE_{se_i}), 0, 0 \end{bmatrix} \quad (4.101)$$

Some of the first and second terms of the Newton equation of the nonlinear interior point OPF are given in the Appendix of this chapter.

4.3.3.5 Implementation of Multi-Configurations and Multi-Control Functions of GUPFC

Multi-configurations of GUPFC. The GUPFC may have the configurations or topologies such as GUPFC or SSSCs plus STATCOM.

For the GUPFC configuration, the operation of GUPFC is mainly constrained by its voltage and current ratings of the converters and the active power exchange of the converters with the DC link. For the SSSCs plus STATCOM configuration the series converters are operated as SSSCs while the shunt converter is operated as a STATCOM, there is active power exchange between the SSSCs and STATCOM. The control configuration is simulated by setting the power exchange limits to zero.

Multi-control functions of GUPFC. As discussed in chapter 3, there are a number of control objectives that can be achieved by the series and shunt control. For instance, in the implementation of other series control functions other than the active and reactive power flow control, the latter can be simply replaced by the new control equations. For the case of power flow control, the following control modes may be adopted:

1. Active and reactive power flow control.
2. Active power flow control only.
3. Reactive power flow control only.
4. Without explicit active and reactive power control objectives.

For the implementation of 1, the dual variable of the reactive power flow control constraint should be dummied in the Newton equation of (4.94). Similarly by dummied the relevant dual variable in the Newton equation, the corresponding control equation can be removed from the equation.

Noting the fact that an OPF is to optimize the system globally, the control mode 4. is more practical and useful. However, for power flow analysis, the active and reactive power flow control equations must be retained.

4.3.3.6 Initialization of GUPFC Variables in Nonlinear Interior OPF

If the GUPFC has explicit series control objectives, then the initialization of the series converters can be done in the same way as discussed in chapter 3 for the GUPFC in power flow calculations. However, if there are no explicit objectives applied, then the injected series voltage magnitude may be set to a value, say $V_{se_{in}} = 0.1 \text{ p.u.}$ ($n=j, k$) while V_{sh_i} is given by:

$$Vsh_i = (Vsh_i^{\max} + Vsh_i^{\min}) / 2 \text{ or } Vsh_i = V_i^{Spec} \quad (4.102)$$

4.3.4 Modeling of IPFC in Nonlinear Interior Point OPF

In comparison to the GUPFC, the IPFC has no shunt converter and the associated control. For the IPFC shown in Fig. 3.4 and Fig. 3.5, the primary series converter $i-j$ has two control degrees of freedom while the secondary series converter $i-k$ has one control degree of freedom since another control degree of freedom of the converter is used to balance the active power exchange between the two series converters. Very similar to that for the GUPFC, A reduced Newton equation for the IPFC can be derived as follows:

$$\left[\begin{array}{c|c} \mathbf{A} & \mathbf{C} \\ \hline \mathbf{C}^T & \mathbf{B} \end{array} \right] \left[\begin{array}{c} \Delta \mathbf{x}^{supfc} \\ \Delta \mathbf{x}^{sys} \end{array} \right] = \left[\begin{array}{c} \mathbf{a} \\ \mathbf{b} \end{array} \right] \quad (4.103)$$

where

$\Delta \mathbf{X}^{ipfc} = [\Delta \mathbf{X}_{ij}^{ipfc}, \Delta \mathbf{X}_{ik}^{ipfc}]^T$ - the incremental vector of the IPFC variables, and

$\Delta \mathbf{X}_{ij}^{ipfc} = [\Delta \mu lse_{ji}, \Delta \theta se_{ij}, \Delta V se_{ij}, \Delta P E se_{ij}, \Delta \lambda P E se_{ij}, \Delta \lambda P se_{ji}, \Delta \lambda Q se_{ji}]^T$ - the incremental vector of the variables of the IPFC primary series branch ij .

$\Delta \mathbf{X}_{ik}^{ipfc} = [\Delta \mu lse_{ik}, \Delta \theta se_{ik}, \Delta V se_{ik}, \Delta P E se_{ik}, \Delta \lambda P E se_{ik}, \Delta \lambda P se_{ki}, \Delta \lambda P Ex]^T$ - the incremental vector of the variables of the IPFC secondary series branch ik .

$\Delta \mathbf{X}^{sys} = [\Delta \mathbf{X}_i^{sys}, \Delta \mathbf{X}_j^{sys}, \Delta \mathbf{X}_k^{sys}]^T$ - the incremental vector of the variables of the system buses.

$\mathbf{a} = [\mathbf{a}_{ij}, \mathbf{a}_{ik}]^T$ - the right hand vector of the IPFC

$\Delta \mathbf{X}_m^{sys} = [\Delta \theta_m, \Delta V_m, \Delta \lambda p_m, \Delta \lambda q_m]^T$ ($m = i, j, k$) - the incremental vector of the variables of system bus m .

In (4.103), all the slack and dual variables of the simple variable inequalities have been eliminated from the formulation. $\mathbf{B}$ and $\mathbf{b}$ are the system matrix and right hand vector, which have similar structure to the system matrix and right hand of (4.54), respectively except that in calculating the former, the contributions from the IPFC should be considered. $\mathbf{a}_{ij}$ and $\mathbf{a}_{ik}$ are given by:

$$\mathbf{a}_{ij} = \begin{bmatrix} -\nabla_{\pi Ise_{ij}} L_{\mu} - (\pi Ise_{ij})^{-1} \nabla_{SuIse_{ij}} L_{\mu} \\ -\nabla_{\theta se_{ij}} L_{\mu} \\ -\nabla_{Vse_{ij}} L_{\mu} + \mu(1/SIVse_{ij} - 1/SuVse_{ij}) \\ -\nabla_{PEse_{ij}} L_{\mu} + \mu(1/SIPEse_{ij} - 1/SuPEse_{ij}) \\ -\nabla_{\lambda PEse_{ij}} L_{\mu} \\ -\nabla_{\lambda Pse_{ji}} L_{\mu} \\ -\nabla_{\lambda Qse_{ji}} L_{\mu} \end{bmatrix} \quad (4.104)$$

$$\mathbf{a}_{ik} = \begin{bmatrix} -\nabla_{\pi Ise_{ik}} L_{\mu} - (\pi Ise_{ik})^{-1} \nabla_{SuIse_{ik}} L_{\mu} \\ -\nabla_{\theta se_{ik}} L_{\mu} \\ -\nabla_{Vse_{ik}} L_{\mu} + \mu(1/SIVse_{ik} - 1/SuVse_{ik}) \\ -\nabla_{PEse_{ik}} L_{\mu} + \mu(1/SIPEse_{ik} - 1/SuPEse_{ik}) \\ -\nabla_{\lambda PEse_{ik}} L_{\mu} \\ -\nabla_{\lambda Pse_{ki}} L_{\mu} \\ -\nabla_{\lambda PEX} L_{\mu} \end{bmatrix} \quad (4.105)$$

Similar to that of the GUPFC, $\mathbf{A}$ and $\mathbf{C}$ are given by:

$$\mathbf{A} = \left[\frac{\partial^2 L}{\partial \mathbf{X}^{ipfc} \partial \mathbf{X}^{ipfc}} \right] + \mathbf{D} \quad (4.106)$$

$$\mathbf{C} = \left[\frac{\partial^2 L}{\partial \mathbf{X}^{ipfc} \partial \mathbf{X}^{sys}} \right] \quad (4.107)$$

where $\mathbf{D}$ is given by

$$\mathbf{D} = \text{Diag}[\mathbf{d}_{ij}, \mathbf{d}_{ik}] \quad (4.108)$$

$$\mathbf{d}_{ij} = \text{Diag} \begin{bmatrix} 0, 0, (\pi Vse_{ij} / SIVse_{ij} - \pi I Vse_{ij} / SuVse_{ij}) \\ (\pi PEse_{ij} / SIPEse_{ij} - \pi I PEse_{ij} / SuPEse_{ij}), 0, 0, 0 \end{bmatrix} \quad (4.109)$$

$$\mathbf{d}_{ik} = \text{Diag} \begin{bmatrix} 0, 0, (\pi Vse_{ik} / SIVse_{ik} - \pi I Vse_{ik} / SuVse_{ik}) \\ (\pi PEse_{ik} / SIPEse_{ik} - \pi I PEse_{ik} / SuPEse_{ik}), 0, 0, 0 \end{bmatrix} \quad (4.110)$$

4.4 Modeling of Multi-Terminal VSC-HVDC in OPF

4.4.1 Multi-Terminal VSC-HVDC in Optimal Voltage and Power Flow

The multi-terminal VSC-HVDC models for power flow analysis have been presented in chapter 3. A multi-terminal VSC-HVDC model suitable for optimal power flow analysis will be discussed here. The multi-terminal VSC-HVDC has not only power flow but also voltage control capability. It is useful to compare the GUPFC with the M-VSC-HVDC and investigate different control capabilities of these two FACTS-devices.

The equivalent circuit of the multi-terminal VSC-HVDC (M-VSC-HVDC) is shown in Fig. 4.2. In this figure, for the sake of simplicity, the VSC HVDC consists of three terminals. However, the derivation is applicable to a M-VSC-HVDC with any number of terminals.

As discussed in chapter 3, the M-VSC-HVDC combining three or more converters working together extends the concepts of voltage and power flow control beyond what is achievable with the known two-converter VSC-HVDC-device. The simplest M-VSC-HVDC consists of three converters connected in shunt with two buses in a substation. It can control total five power system quantities such as a bus voltage of the secondary converter and independent active and reactive power flows of two lines, which are connected with the primary converters.

In the steady state operation, the main objective of the M-VSC-HVDC is to control voltage and power flow. Real power can be exchanged among these converters via the common DC link. The sum of the real power exchange should be zero if we neglect the losses of the converter circuits. For the M-VSC-HVDC shown in Fig. 4.2, if explicit controls are applied, it has total 5 degrees of control freedom, that means it can control five power system quantities such as one bus voltage, and 4 active and reactive power flows of two lines. It can be seen that with more converters included within the M-VSC-HVDC, more degrees of control freedom can be introduced and hence more control objectives can be achieved.

4.4.2 Operating and Control Constraints of the M-VSC-HVDC

In Fig. 4.2, the AC power flow constraints of the M-VSC-HVDC at buses i, j, k can be explicitly incorporated into the power mismatch equations at these AC buses.

The active power exchange among the converters via the DC link should be balanced at any instant, which is described by:

$$PEx = Pdc_i + Pdc_j + Pdc_k + Ploss = 0 \quad (4.111)$$

where $Ploss$ represents losses in converter circuits. The handling of $Ploss$ has been discussed in chapter 3.

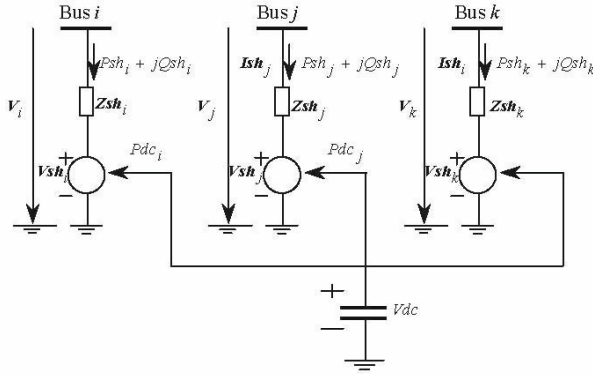


Fig. 4.2. The equivalent circuit of the multi-terminal VSC HVDC

Pdc_m ($m = i, j, k$) as shown in Fig. 4.2 is the power exchange of the converter with the DC link and can be described by the following equalities:

$$Pdc_m - \operatorname{Re}(-Vsh_m Ish_m^*) = 0 \quad m = i, j, k \quad (4.112)$$

where

$$\operatorname{Re}(-Vsh_m Ish_m^*) = Vsh_m^2 gsh_m - V_i Vsh_m (gsh_m \cos(\theta_i - \theta_{sh_m}) - bsh_m \sin(\theta_i - \theta_{sh_m})).$$

Voltage and power flow control constraints of the M-VSC-HVDC consists of explicit PQ or PV control of primary converters as given by (3.79)-(3.82) and voltage control of secondary converter. In the implementation of the voltage control, the equality is simply replaced by an inequality of bus voltage constraint since the implementation of the simple variable inequality constraint is very simple and straightforward.

In addition to the above equality constraints, voltage and current inequality constraints of the M-VSC-HVDC as shown in (3.84) and (3.85) should be considered.

4.4.3 Modeling of M-VSC-HVDC in the Nonlinear Interior Point OPF

Following the similar procedure in the derivation of Nonlinear Interior Point OPF with incorporation of the GUPFC, a reduced Newton equation can be obtained:

With the incorporation of the augmented Lagrangian functions above into the OPF problem in section 4.2, a reduced Newton equation can be derived:

$$\begin{bmatrix} \mathbf{A} & \mathbf{C} \\ \mathbf{C}^T & \mathbf{B} \end{bmatrix} \begin{bmatrix} \Delta \mathbf{x}^{HVDC} \\ \Delta \mathbf{x}^{sys} \end{bmatrix} = \begin{bmatrix} \mathbf{a} \\ \mathbf{b} \end{bmatrix} \quad (4.113)$$

where

$\Delta \mathbf{X}^{HVDC} = [\Delta \mathbf{X}_i^{HVDC}, \Delta \mathbf{X}_j^{HVDC}, \Delta \mathbf{X}_k^{HVDC}]^T$ - the incremental vector of the M-VSC-HVDC variables, and

$\Delta \mathbf{X}_i^{HVDC} = [\Delta \pi I_{sh_i}, \Delta \theta_{sh_i}, \Delta V_{sh_i}, \Delta P_{Esh_i}, \Delta \lambda P_{Esh_i}, \Delta \lambda P_{Ex}]^T$ - the incremental vector of the variables of the M-VSC-HVDC branch i .

$\Delta \mathbf{X}_j^{HVDC} = [\Delta \pi I_{sh_j}, \Delta \theta_{sh_j}, \Delta V_{sh_j}, \Delta P_{Esh_j}, \Delta \lambda P_{Esh_j}, \Delta \lambda P_{Esh_j}, \lambda Q_{Esh_j}]^T$ - the incremental vector of the variables of the M-VSC-HVDC branch j .

$\Delta \mathbf{X}_k^{HVDC} = [\Delta \pi I_{sh_k}, \Delta \theta_{sh_k}, \Delta V_{sh_k}, \Delta P_{Esh_k}, \Delta \lambda P_{Esh_k}, \Delta \lambda P_{Esh_k}, \lambda Q_{Esh_k}]^T$ the incremental vector of the variables of the M-VSC-HVDC branch k .

$\Delta \mathbf{X}^{sys} = [\Delta \mathbf{X}_i^{sys}, \Delta \mathbf{X}_j^{sys}, \Delta \mathbf{X}_k^{sys}]^T$ - the incremental vector of the variables of the system buses.

$\mathbf{a} = [\mathbf{a}_i, \mathbf{a}_j, \mathbf{a}_k]^T$ - the right hand vector of the M-VSC-HVDC.

$\Delta \mathbf{X}_m^{sys} = [\Delta \theta_m, \Delta V_m, \Delta \lambda p_m, \Delta \lambda q_m]^T$ ($m = i, j, k$) - the incremental vector of the variables of system bus m .

In (4.113), all the slack and dual variables of the simple variable inequalities have been eliminated from the formulation. $\mathbf{B}$ and $\mathbf{b}$ are the system matrix and right hand vector have similar structure to the system matrix and right hand of (4.54), respectively except that in calculating the former, the contributions from the M-VSC-HVDC should be considered. $\mathbf{a}_i$ and $\mathbf{a}_m$ are given by:

$$\mathbf{a}_i = \begin{bmatrix} -\nabla_{\pi I_{sh_i}} L_\mu - (\pi I_{sh_i})^{-1} \nabla_{Su I_{sh_i}} L_\mu \\ -\nabla_{\theta_{sh_i}} L_\mu \\ -\nabla_{V_{sh_i}} L_\mu + \mu(1/SI V_{sh_i} - 1/Su V_{sh_i}) \\ -\nabla_{P_{Esh_i}} L_\mu + \mu(1/SI P_{Esh_i} - 1/Su P_{Esh_i}) \\ -\nabla_{\lambda P_{Esh_i}} L_\mu \\ -\nabla_{\lambda P_{Ex}} L_\mu \end{bmatrix} \quad (4.114)$$

$$\mathbf{a}_m = \begin{bmatrix} -\nabla_{\pi I_{sh_m}} L_\mu - (\pi I_{sh_m})^{-1} \nabla_{Su I_{sh_m}} L_\mu \\ -\nabla_{\theta_{sh_m}} L_\mu \\ -\nabla_{V_{sh_m}} L_\mu + \mu(1/SI V_{sh_m} - 1/Su V_{sh_m}) \\ -\nabla_{P_{Esh_m}} L_\mu + \mu(1/SI P_{Esh_m} - 1/Su P_{Esh_m}) \\ -\nabla_{\lambda P_{Esh_m}} L_\mu \\ -\nabla_{\lambda P_{sh_m}} L_\mu \\ -\nabla_{\lambda Q_{sh_m}} L_\mu \end{bmatrix} \quad (m = j, k) \quad (4.115)$$

In (4.113), $\mathbf{A}$ and $\mathbf{C}$ are given by:

$$\mathbf{A} = \left[\frac{\partial^2 L}{\partial \mathbf{X}^{HVDC} \partial \mathbf{X}^{HVDC}} \right] + \mathbf{D} \quad (4.116)$$

$$\mathbf{C} = \left[\frac{\partial^2 L}{\partial \mathbf{X}^{HVDC} \partial \mathbf{X}^{sys}} \right] \quad (4.117)$$

where $\mathbf{D}$ is given by:

$$\mathbf{D} = \text{Diag}[\mathbf{d}_i, \mathbf{d}_j, \mathbf{d}_k] \quad (4.118)$$

$$\mathbf{d}_i = \text{Diag} \left[0, 0, (\pi V_{sh_i} / SIV_{sh_i} - \pi u V_{sh_i} / SuV_{sh_i}), (\pi P_{Esh_i} / SIPE_{sh_i} - \pi u P_{Esh_i} / SuPE_{sh_i}), 0, 0 \right] \quad (4.119)$$

$$\mathbf{d}_m = \text{Diag} \left[0, 0, (\pi V_{sh_m} / SIV_{sh_m} - \pi u V_{sh_m} / SuV_{sh_m}), (\pi P_{Esh_m} / SIPE_{sh_m} - \pi u P_{Esh_m} / SuPE_{sh_m}), 0, 0, 0 \right] \quad (4.120)$$

$(m = j, k)$

4.5 Comparison of FACTS-Devices with VSC-HVDC

The test cases in this section are carried out on the IEEE 30 bus system and IEEE118-bus system. The IEEE 30-bus system has 6 generators, 4 OLTC transformers and 37 transmission lines. The IEEE 118-bus system has 18 controllable active power generation, 54 controllable reactive power generation, 9 OLTC transformers, 177 transmission lines. For all cases in this section, the convergence tolerances are 5.0×10^{-4} for complementary gap and 1.0×10^{-4} (0.01MW/Mvar) for maximal absolute bus power mismatch, respectively.

4.5.1 Comparison of UPFC with BTB-VSC-HVDC

As a special case, Back-to-Back (BTB) VSC-HVDC should have similar a control capability as UPFC. Cases, which are used to show the control performance of both BTB-VSC-HVDC and UPFC, are presented. Four cases are given on the IEEE 30-bus system:

Case 1: A UPFC is installed for control of the voltage at bus 12 and control of active and reactive power flows in line 12-15. Suppose the FACTS bus 15 is created, and assume that the sending end of the transmission line 12-15 is now connected with the FACTS buses 15' while the series converter is

installed between buses 12 and 15.

Case 2: Similar to case 1, but no explicit UPFC control is applied.

Case 3: A BTB VSC-HVDC is used to replace the UPFC in case 1 while the control settings for voltage and power flows are as the same as that of case 1. This also means the two primary converters are using the PQ control mode.

Case 4: Similar to case 3, but no BTB-VSC-HVDC is applied.

The transformer impedance is $0 + j0.025$ p.u. The active power flow settings on the transmission line 12-15 are 25 MW and 10 MVar, respectively, for active and reactive power flows while voltage control setting is 1.0 p.u.

For cases 1 and 2 and cases 3 and 4, the results have been obtained for two different situations: (a) there are explicit voltage and power flow control objectives; (b) there are no explicit voltage and power flow control objectives. For the second situation, global optimization of control settings of voltage and power flows of the FACTS-devices is expected. Table 4.2 and Table 4.3 show the test results.

From Table 4.2, it can be seen that the OPF solution with UPFC needs more iterations in comparison to that with BTB-VSC-HVDC. Furthermore, UPFC cases without explicit control will need more iterations than that with explicit control. In contrast, it has been found that the OPF algorithm with BTB-VSC-HVDC is not sensitive to the initial point and can converge more quickly.

Table 4.2. Optimal power flow results for the IEEE 30-bus system with UPFC or BTB VSC HVDC

Case No.	Case 1	Case 2	Case 3	Case 4
Control type	With explicit FACTS control	Without explicit FACTS control	With explicit FACTS control	Without explicit FACTS control
Objective function	8.0413e+002	8.0344e+002	8.0413e+002	8.0344e+002
Active power loss	9.97 MW	9.76 MW	9.97 MW	9.76 MW
Total active power generation	293.37 MW	293.16 MW	293.37 MW	293.16 MW
Total reactive power generation	127.84 MVar	112.52 Mvar	129.78 MVar	112.08 MVar
Number of iterations	11	17	9	9

Table 4.3. The OPF solution of UPFC or BTB VSC HVDC for the IEEE 30-bus system

Case No.	Case 1	Case 2	Case 3	Case 4
Control type	With explicit FACTS control	Without explicit FACTS control	With explicit FACTS control	Without explicit FACTS control
FACTS solution	UPFC: $V_{sh_{12,15'}} = 0.0435$ p.u. $\theta_{se_{12,15'}} = 151.94^\circ$ $V_{sh_{12}} = 1.002$ p.u. $\theta_{sh_{12}} = 11.70^\circ$	UPFC: $V_{sh_{12,15'}} = 0.00549$ p.u. $\theta_{se_{12,15'}} = -147.35^\circ$ $V_{sh_{12}} = 1.048$ p.u. $\theta_{sh_{12}} = -11.21^\circ$	BTB HVDC: $V_{sh_{15'}} = 1.034$ p.u. $\theta_{sh_{15'}} = -10.15^\circ$ $V_{sh_{12}} = 1.000$ p.u. $\theta_{sh_{12}} = -12.07^\circ$	BTB HVDC: $V_{sh_{15'}} = 1.046$ p.u. $\theta_{sh_{15'}} = -11.45^\circ$ $V_{sh_{12}} = 1.047$ p.u. $\theta_{sh_{12}} = -11.01^\circ$
FACTS converter max. power	UPFC: <i>Series converter:</i> <i>1.1 MVA</i> <i>Shunt converter:</i> <i>5.4 MVA</i>	UPFC: <i>Series converter:</i> <i>0.1 MVA</i> <i>Shunt converter:</i> <i>19 MVA</i>	BTB HVDC: <i>Primary conv.:</i> <i>27 MVA</i> <i>Secondary conv.:</i> <i>26 MVA</i>	BTB HVDC: <i>Primary conv.:</i> <i>19 MVA</i> <i>Secondary conv.:</i> <i>22 MVA</i>

From Table 4.3, it can be seen that using UPFC and BTB VSC HVDC can achieve the similar control purposes. However, ratings of the converters are quite different. In the studies, it has been found that the rating of the series converter of a UPFC is very small. This means a reduced investment in comparison to a BTB VSC HVDC.

4.5.2 Comparison of GUPFC with M-VSC-HVDC

Case studies are carried out on the IEEE 118-bus system with GUPFC installed. Five cases are presented as follows:

Case 5: The base case of the IEEE 118-bus system.

Case 6: There is a GUPFC installed for control of voltage at bus 45 and active and reactive power flow of line 45-44 and line 45-46. The control setting of the bus voltage is 1.0p.u. The control settings for active and reactive power flow of line 45-44 and line 45-46 are 40MW +j7Mvar and -50MW-j7Mvar, respectively. There is second GUPFC further installed for control voltage of bus 94 and power flow of line 94-95, line 94-93, line 94-100. The voltage control objective is 1.0 p.u. The control settings for active and reactive power flow of line 94-95, line 94-93, line 94-100 are 50MW +j5Mvar, -50MW-j20Mvar and -35MW -j10Mvar, respectively. There is third GUPFC further installed for control voltage of bus 12 and power flow of line 12-3 and line 12-11. The control settings for active and reactive power flow of line 12-3, line 12-11 are 15MW +j4Mvar and -40 MW + j15Mvar, respectively.

Case 7: This is similar to case 6 except that there are no explicit FACTS controls applied.

The following cases are carried out on the IEEE 118-bus system with M-VSC-HVDC installed:

Case 8: This is similar to case 6 except that all GUPFC-devices are replaced by M-VSC-HVDCs.

Case 9: This is similar to case 8 except that there are no explicit M-VSC-HVDC controls applied.

Test results based on the cases 5-9 are summarized in Table 4.4. In these cases, active power flow settings are over 125% of their corresponding base case active power flows.

The power flow solutions of case 6 and case 8 for the GUPFCs and M-VSC-HVDCs are shown in Table 4.5. As expected, multi-terminal VSC HVDCs have similar steady state control capability as that of GUPFCs. However, it has been found that the actual power of a converter of GUPFC is much less than that of a converter of M-VSC-HVDC.

Table 4.4. Test results of the IEEE 118-bus system

	Case 5	Case 6	Case 7	Case 8	Case 9
FACTS type	None	GUPFC	GUPFC	M-VSC-HVDC	M-VSC-HVDC
Control type	None	With explicit control	Without explicit control	With explicit control	Without explicit control
Number of devices	None	3	3	3	3
Total number of active and reactive power flow control	None	7P Flow 7Q Flow	0	7P Flow 7Q Flow	0
Total number of voltage control	None	3	0	3	0
Number of iterations	12	14	21	12	12

Table 4.5. FACTS-solutions of case 6 and case 8

	Case 6	Case 8
FACTS type	GUPFC	M-VSC-HVDC
Control type	With explicit control	With explicit control
Number of devices	3	3
Maximum power of converter	GUPFC at bus 12 <i>Series converter: 0.6 MVA</i> <i>Series converter : 0.01 MVA</i> <i>Shunt converter: 19.7 MVA</i>	M-VSC-HVDC at bus 12 <i>Primary converter: 21.2 MVA</i> <i>Primary converter: 25.1 MVA</i> <i>Secondary converter: 20.9 MVA</i>
	GUPFC at bus 45 <i>Series converter 1: 2.4 MVA</i> <i>Series converter 2: 2.6 MVA</i> <i>Shunt converter: 10 MVA</i>	M-VSC-HVDC at bus 45 <i>Primary converter 1: 40.6 MVA</i> <i>Primary converter 2: 50.2 MVA</i> <i>Secondary converter: 25.2 MVA</i>
	GUPFC at bus 94 <i>Series converter 1: 0.4 MVA</i> <i>Series converter 2: 2.9 MVA</i> <i>Series converter 3: 2.9 MVA</i> <i>Shunt converter: 49.7 MVA</i>	M-VSC-HVDC at bus 94 <i>Primary converter 1: 50.3 MVA</i> <i>Primary converter 2: 53.6 MVA</i> <i>Primary converter 3: 36.3 MVA</i> <i>Secondary converter: 52.9 MVA</i>

From these results on the IEEE 30-bus and 118-bus systems, it can be seen:

1. Numerical results demonstrate the feasibility as well as the effectiveness of the FACTS and VSC-HVDC models established and the OPF method proposed.
2. The GUPFC and the M-VSC-HVDC are quite flexible and powerful FACTS-device. Both of them can control bus voltage and active and reactive power flows of several lines simultaneously. They may be installed in some central substations to manage power flows of multi-lines or a group of lines and provide voltage support as well.
3. The OPF with global coordinating capability is a very useful tool to minimize (or maximize) an objective while satisfying power flow constraints, thermal constraints, as well as the operating and control constraints of the GUPFC devices.
4. The flexibility of the GUPFC and the M-VSC-HVDC with controlling bus voltage and multi-line active and reactive power flows offers a great potential in solving many of the problems facing the electric utilities in a competitive environment.

5. The power rating of a primary converter of the M-VSC-HVDC may be higher than that of a corresponding series converter of the GUPFC since the voltage rating of the former is higher than that of the latter. Hence, the power rating of the secondary converter of the M-VSC-HVDC may be higher than that of the shunt converter of the GUPFC.
6. VSC-HVDC and UPFC, and M-VSC-HVDC and GUPFC may be used interchangeably. However, the investment of the former may be higher than that of the latter.

4.6 Appendix: Derivatives of Nonlinear Interior Point OPF with GUPFC

The power mismatches at bus m are ΔP_m , ΔQ_m .

$$\Delta P_m = Pg_m - Pd_m - P_m \quad (4.121)$$

$$\Delta Q_m = Qg_m - Qd_m - Q_m \quad (4.122)$$

where P_m and Q_m are the sum of the active power flow and reactive power flow at bus m , respectively.

4.6.1 First Derivatives of Nonlinear Interior Point OPF

$$\frac{\partial P_i}{\partial \theta se_{ij}} = V_i V se_{ij} (g_{ij} \sin(\theta_i - \theta se_{ij}) - b_{ij} \cos(\theta_i - \theta se_{ij})) \quad (4.123)$$

$$\frac{\partial P_i}{\partial V se_{ij}} = -V_i (g_{ij} \cos(\theta_i - \theta se_{ij}) + b_{ij} \sin(\theta_i - \theta se_{ij})) \quad (4.124)$$

$$\frac{\partial P_i}{\partial \theta sh_i} = -V_i V sh_i (g sh_i \sin(\theta_i - \theta sh_i) - b sh_i \cos(\theta_i - \theta sh_i)) \quad (4.125)$$

$$\frac{\partial P_i}{\partial V sh_i} = -V_i (g sh_i \cos(\theta_i - \theta sh_i) + b sh_i \sin(\theta_i - \theta sh_i)) \quad (4.126)$$

$$\frac{\partial Q_i}{\partial \theta se_{ij}} = V_i V se_{ij} (g_{ij} \cos(\theta_i - \theta se_{ij}) + b_{ij} \sin(\theta_i - \theta se_{ij})) \quad (4.127)$$

$$\frac{\partial Q_i}{\partial V se_{ij}} = -V_i (g_{ij} \sin(\theta_i - \theta se_{ij}) - b_{ij} \cos(\theta_i - \theta se_{ij})) \quad (4.128)$$

$$\frac{\partial Q_i}{\partial \theta sh_i} = V_i V sh_i (g sh_i \cos(\theta_i - \theta sh_i) + b sh_i \sin(\theta_i - \theta sh_i)) \quad (4.129)$$

$$\frac{\partial Q_i}{\partial Vsh_i} = -V_i(gsh_i \sin(\theta_i - \theta sh_i) - bsh_i \cos(\theta_i - \theta sh_i)) \quad (4.130)$$

$$\frac{\partial P_j}{\partial \theta se_{ij}} = V_j Vse_{ij}(g_{ij} \sin(\theta_j - \theta se_{ij}) - b_{ij} \cos(\theta_j - \theta se_{ij})) \quad (4.131)$$

$$\frac{\partial P_j}{\partial Vse_{ij}} = -V_j(g_{ij} \cos(\theta_j - \theta se_{ij}) + b_{ij} \sin(\theta_j - \theta se_{ij})) \quad (4.132)$$

$$\frac{\partial Q_j}{\partial \theta se_{ij}} = -V_j Vse_{ij}(g_{ij} \cos(\theta_j - \theta se_{ij}) + b_{ij} \sin(\theta_j - \theta se_{ij})) \quad (4.133)$$

$$\frac{\partial Q_j}{\partial Vse_{ij}} = V_j(g_{ij} \sin(\theta_j - \theta se_{ij}) - b_{ij} \cos(\theta_j - \theta se_{ij})) \quad (4.134)$$

$$\begin{aligned} \frac{\partial(Vse_{ij} I_{ji}^*)}{\partial \theta se_{ij}} &= -V_i Vse_{ij}(g_{ij} \sin(\theta se_{ij} - \theta_i) - b_{ij} \cos(\theta se_{ij} - \theta_i)) \\ &\quad + V_j Vse_{ij}(g_{ij} \sin(\theta se_{ij} - \theta_j) - b_{ij} \cos(\theta se_{ij} - \theta_j)) \end{aligned} \quad (4.135)$$

$$\begin{aligned} \frac{\partial(Vse_{ij} I_{ji}^*)}{\partial Vse_{ij}} &= -2g_{ij} Vse_{ij} + V_i(g_{ij} \cos(\theta se_{ij} - \theta_i) + b_{ij} \sin(\theta se_{ij} - \theta_i)) \\ &\quad - V_j(g_{ij} \cos(\theta se_{ij} - \theta_j) + b_{ij} \sin(\theta se_{ij} - \theta_j)) \end{aligned} \quad (4.136)$$

$$\frac{\partial(Vse_{ij} I_{ji}^*)}{\partial \theta_i} = V_i Vse_{ij}(g_{ij} \sin(\theta se_{ij} - \theta_i) - b_{ij} \cos(\theta se_{ij} - \theta_i)) \quad (4.137)$$

$$\frac{\partial(Vse_{ij} I_{ji}^*)}{\partial V_i} = Vse_{ij}(g_{ij} \cos(\theta se_{ij} - \theta_i) + b_{ij} \sin(\theta se_{ij} - \theta_i)) \quad (4.138)$$

$$\frac{\partial(Vse_{ij} I_{ji}^*)}{\partial \theta_j} = -V_j Vse_{ij}(g_{ij} \sin(\theta se_{ij} - \theta_j) - b_{ij} \cos(\theta se_{ij} - \theta_j)) \quad (4.139)$$

$$\frac{\partial(Vse_{ij} I_{ji}^*)}{\partial V_j} = -Vse_{ij}(g_{ij} \cos(\theta se_{ij} - \theta_j) + b_{ij} \sin(\theta se_{ij} - \theta_j)) \quad (4.140)$$

$$\frac{\partial(Vsh_i Ish_i^*)}{\partial \theta sh_i} = -V_i Vsh_i(gsh_i \sin(\theta sh_i - \theta_i) - bsh_i \cos(\theta sh_i - \theta_i)) \quad (4.141)$$

$$\frac{\partial(Vsh_i Ish_i^*)}{\partial Vsh_i} = 2gsh_i Vsh_i - V_i(gsh_i \cos(\theta sh_i - \theta_i) + bsh_i \sin(\theta sh_i - \theta_i)) \quad (4.142)$$

$$\frac{\partial(Vsh_i Ish_i^*)}{\partial \theta_i} = -V_i Vsh_i(gsh_i \sin(\theta sh_i - \theta_i) - bsh_i \cos(\theta sh_i - \theta_i)) \quad (4.143)$$

$$\frac{\partial(Vsh_i Ish_i^*)}{\partial V_i} = -Vsh_i(gsh_i \cos(\theta sh_i - \theta_i) + bsh_i \sin(\theta sh_i - \theta_i)) \quad (4.144)$$

4.6.2 Second Derivatives of Nonlinear Interior Point OPF

$$\frac{\partial^2 P_i}{\partial^2 \theta e_{ij}} = V_i V s e_{ij} (g_{ij} \cos(\theta_i - \theta e_{ij}) + b_{ij} \sin(\theta_i - \theta e_{ij})) \quad (4.145)$$

$$\frac{\partial^2 P_i}{\partial \theta e_{ij} \partial V s e_{ij}} = -V_i (g_{ij} \sin(\theta_i - \theta e_{ij}) - b_{ij} \cos(\theta_i - \theta e_{ij})) \quad (4.146)$$

$$\frac{\partial^2 P_i}{\partial^2 V s e_{ij}} = 0 \quad (4.147)$$

$$\frac{\partial^2 P_i}{\partial^2 \theta h_i} = V_i V s h_i (g s h_i \cos(\theta_i - \theta h_i) + b s h_i \sin(\theta_i - \theta h_i)) \quad (4.148)$$

$$\frac{\partial^2 P_i}{\partial \theta h_i \partial V s h_i} = -V_i (g s h_i \sin(\theta_i - \theta h_i) - b s h_i \cos(\theta_i - \theta h_i)) \quad (4.149)$$

$$\frac{\partial^2 P_i}{\partial^2 V s h_i} = 0 \quad (4.150)$$

$$\frac{\partial^2 Q_i}{\partial^2 \theta e_{ij}} = V_i V s e_{ij} (g_{ij} \sin(\theta_i - \theta e_{ij}) - b_{ij} \cos(\theta_i - \theta e_{ij})) \quad (4.151)$$

$$\frac{\partial^2 Q_i}{\partial \theta e_{ij} \partial V s e_{ij}} = V_i (g_{ij} \cos(\theta_i - \theta e_{ij}) + b_{ij} \sin(\theta_i - \theta e_{ij})) \quad (4.152)$$

$$\frac{\partial^2 Q_i}{\partial^2 V s e_{ij}} = 0 \quad (4.153)$$

$$\frac{\partial^2 Q_i}{\partial^2 \theta h_i} = V_i V s h_i (g s h_i \sin(\theta_i - \theta h_i) - b s h_i \cos(\theta_i - \theta h_i)) \quad (4.154)$$

$$\frac{\partial^2 Q_i}{\partial \theta h_i \partial V s h_i} = V_i (g s h_i \cos(\theta_i - \theta h_i) + b s h_i \sin(\theta_i - \theta h_i)) \quad (4.155)$$

$$\frac{\partial^2 \Delta Q_i}{\partial^2 V s h_i} = 0 \quad (4.156)$$

$$\frac{\partial^2 P_j}{\partial^2 \theta e_{ij}} = -V_j V s e_{ij} (g_{ij} \cos(\theta_j - \theta e_{ij}) + b_{ij} \sin(\theta_j - \theta e_{ij})) \quad (4.157)$$

$$\frac{\partial^2 P_j}{\partial \theta e_{ij} \partial V s e_{ij}} = V_j (g_{ij} \sin(\theta_j - \theta e_{ij}) - b_{ij} \cos(\theta_j - \theta e_{ij})) \quad (4.158)$$

$$\frac{\partial^2 P_j}{\partial^2 Vse_{ij}} = 0 \quad (4.159)$$

$$\frac{\partial^2 Q_j}{\partial^2 \theta se_{ij}} = -V_j Vse_{ij} (g_{ij} \sin(\theta_j - \theta se_{ij}) - b_{ij} \cos(\theta_j - \theta se_{ij})) \quad (4.160)$$

$$\frac{\partial^2 Q_j}{\partial \theta se_{ij} \partial Vse_{ij}} = -V_j (g_{ij} \cos(\theta_j - \theta se_{ij}) + b_{ij} \sin(\theta_j - \theta se_{ij})) \quad (4.161)$$

$$\frac{\partial^2 Q_j}{\partial^2 Vse_{ij}} = 0 \quad (4.162)$$

$$\begin{aligned} \frac{\partial^2 (Vse_{ij} I_{ji}^*)}{\partial^2 \theta se_{ij}} &= -V_i Vse_{ij} (g_{ij} \cos(\theta se_{ij} - \theta_i) + b_{ij} \sin(\theta se_{ij} - \theta_i)) \\ &\quad + V_j Vse_{ij} (g_{ij} \cos(\theta se_{ij} - \theta_j) + b_{ij} \sin(\theta se_{ij} - \theta_j)) \end{aligned} \quad (4.163)$$

$$\begin{aligned} \frac{\partial^2 (Vse_{ij} I_{ji}^*)}{\partial \theta se_{ij} \partial Vse_{ij}} &= -V_i (g_{ij} \sin(\theta se_{ij} - \theta_i) - b_{ij} \cos(\theta se_{ij} - \theta_i)) \\ &\quad + V_j (g_{ij} \sin(\theta se_{ij} - \theta_j) - b_{ij} \cos(\theta se_{ij} - \theta_j)) \end{aligned} \quad (4.164)$$

$$\frac{\partial^2 (Vse_{ij} I_{ji}^*)}{\partial \theta se_{ij} \partial \theta_i} = V_i Vse_{ij} (g_{ij} \cos(\theta se_{ij} - \theta_i) + b_{ij} \sin(\theta se_{ij} - \theta_i)) \quad (4.165)$$

$$\frac{\partial^2 (Vse_{ij} I_{ji}^*)}{\partial \theta se_{ij} \partial V_i} = -Vse_{ij} (g_{ij} \sin(\theta se_{ij} - \theta_i) - b_{ij} \cos(\theta se_{ij} - \theta_i)) \quad (4.166)$$

$$\frac{\partial^2 (Vse_{ij} I_{ji}^*)}{\partial \theta se_{ij} \partial \theta_j} = -V_j Vse_{ij} (g_{ij} \cos(\theta se_{ij} - \theta_j) + b_{ij} \sin(\theta se_{ij} - \theta_j)) \quad (4.167)$$

$$\frac{\partial^2 (Vse_{ij} Ise_{ij}^*)}{\partial \theta se_{ij} \partial V_j} = Vse_{ij} (g_{ij} \sin(\theta se_{ij} - \theta_j) - b_{ij} \cos(\theta se_{ij} - \theta_j)) \quad (4.168)$$

$$\frac{\partial^2 (Vse_{ij} Ise_{ij}^*)}{\partial Vse_{ij} \partial \theta_i} = V_i (g_{ij} \sin(\theta se_{ij} - \theta_i) - b_{ij} \cos(\theta se_{ij} - \theta_i)) \quad (4.169)$$

$$\frac{\partial^2 (Vse_{ij} Ise_{ij}^*)}{\partial Vse_{ij} \partial V_i} = -(g_{ij} \cos(\theta se_{ij} - \theta_i) + b_{ij} \sin(\theta se_{ij} - \theta_i)) \quad (4.170)$$

$$\frac{\partial^2 (Vse_{ij} Ise_{ij}^*)}{\partial Vse_{ij} \partial \theta_j} = -V_j (g_{ij} \sin(\theta se_{ij} - \theta_j) - b_{ij} \cos(\theta se_{ij} - \theta_j)) \quad (4.171)$$

$$\frac{\partial^2 (Vse_{ij} I_{ji}^*)}{\partial Vse_{ij} \partial V_j} = -(g_{ij} \cos(\theta se_{ij} - \theta_j) + b_{ij} \sin(\theta se_{ij} - \theta_j)) \quad (4.172)$$

$$\frac{\partial^2 (\mathbf{Vse}_{ij} \mathbf{I}_{ji}^*)}{\partial^2 \theta_i} = -V_i Vse_{ij} (g_{ij} \cos(\theta se_{ij} - \theta_i) + b_{ij} \sin(\theta se_{ij} - \theta_i)) \quad (4.173)$$

$$\frac{\partial^2 (\mathbf{Vse}_{ij} \mathbf{I}_{ji}^*)}{\partial \theta_i \partial V_i} = Vse_{ij} (g_{ij} \sin(\theta se_{ij} - \theta_i) - b_{ij} \cos(\theta se_{ij} - \theta_i)) \quad (4.174)$$

$$\frac{\partial^2 (\mathbf{Vse}_{ij} \mathbf{I}_{ji}^*)}{\partial^2 \theta_j} = V_j Vse_{ij} (g_{ij} \cos(\theta se_{ij} - \theta_j) + b_{ij} \sin(\theta se_{ij} - \theta_j)) \quad (4.175)$$

$$\frac{\partial^2 (\mathbf{Vse}_{ij} \mathbf{I}_{ji}^*)}{\partial \theta_j \partial V_j} = -Vse_{ij} (g_{ij} \sin(\theta se_{ij} - \theta_j) - b_{ij} \cos(\theta se_{ij} - \theta_j)) \quad (4.176)$$

$$\frac{\partial^2 (\mathbf{Vsh}_i \mathbf{Ish}_i^*)}{\partial^2 \theta sh_i} = -V_i Vsh_i (gsh_i \cos(\theta sh_i - \theta_i) + bsh_i \sin(\theta sh_i - \theta_i)) \quad (4.177)$$

$$\frac{\partial^2 (\mathbf{Vsh}_i \mathbf{Ish}_i^*)}{\partial \theta sh_i \partial Vsh_i} = V_i (gsh_i \sin(\theta sh_i - \theta_i) - bsh_i \cos(\theta sh_i - \theta_i)) \quad (4.178)$$

$$\frac{\partial^2 (\mathbf{Vsh}_i \mathbf{Ish}_i^*)}{\partial \theta sh_i \partial \theta_i} = -V_i Vsh_i (gsh_i \cos(\theta sh_i - \theta_i) + bsh_i \sin(\theta sh_i - \theta_i)) \quad (4.179)$$

$$\frac{\partial^2 (\mathbf{Vsh}_i \mathbf{Ish}_i^*)}{\partial \theta sh_i \partial V_i} = Vsh_i (gsh_i \sin(\theta sh_i - \theta_i) - bsh_i \cos(\theta sh_i - \theta_i)) \quad (4.180)$$

$$\frac{\partial^2 (\mathbf{Vsh}_i \mathbf{Ish}_i^*)}{\partial^2 Vsh_i} = 2.0 gsh_i \quad (4.181)$$

$$\frac{\partial^2 (\mathbf{Vsh}_i \mathbf{Ish}_i^*)}{\partial Vsh_i \partial \theta_i} = -V_i (gsh_i \sin(\theta sh_i - \theta_i) - bsh_i \cos(\theta sh_i - \theta_i)) \quad (4.182)$$

$$\frac{\partial^2 (\mathbf{Vsh}_i \mathbf{Ish}_i^*)}{\partial Vsh_i \partial V_i} = -(gsh_i \cos(\theta sh_i - \theta_i) + bsh_i \sin(\theta sh_i - \theta_i)) \quad (4.183)$$

$$\frac{\partial^2 (\mathbf{Vsh}_i \mathbf{Ish}_i^*)}{\partial^2 \theta_i} = V_i Vsh_i (gsh_i \cos(\theta sh_i - \theta_i) + bsh_i \sin(\theta sh_i - \theta_i)) \quad (4.184)$$

$$\frac{\partial^2 (\mathbf{Vsh}_i \mathbf{Ish}_i^*)}{\partial \theta_i \partial V_i} = -Vsh_i (gsh_i \sin(\theta sh_i - \theta_i) - bsh_i \cos(\theta sh_i - \theta_i)) \quad (4.185)$$

References

- [1] Kirchmayer LK (1958) Economic operation of power systems. John Wiley & Sons, New York
- [2] Kirchmayer LK (1959) Economic control of interconnected systems. John Wiley & Sons, New York
- [3] Carpentier JL (1962) Contribution a. 'l'etude du dispatching economique. Bulletin de la Societe Francaise des Electriciens, vol 3, pp 431-447
- [4] Dommel HW, Tinney WF (1968) Optimal power flow solutions. IEEE Trans. on PAS, vol 87, no 10, pp1866-1876
- [5] Happ HH (1977), Optimal power dispatch – A comprehensive survey. IEEE Transactions on PAS, vol 96, pp 841-854
- [6] Carpentier JL (1985) Optimal power flows: uses, methods and developments, Proceedings of IFAC Conference
- [7] Stott B, Alsac O, Monticelli A (1987) Security and optimization, Proceedings of the IEEE, vol 75, no 12, pp 1623-1624
- [8] Carpentier JL (1987) Towards a secure and optimal automatic operation of power systems. Proceedings of Power Industry Computer Applications (PICA) conference, pp 2-37
- [9] Wu FF (1988) Real-time network security monitoring, assessment and optimization. International Journal of Electrical Power and Energy Systems, vol 10, no 2, pp 83-100
- [10] Chowdhury BH, Rahman S (1990) A review of recent advances in economic dispatch. IEEE Transactions on Power Systems, vol 5, no 4, pp 1248-1257
- [11] Huneault M, Galiana FD (1991) A survey of the optimal power flow literature. IEEE Transactions on Power Systems, vol 6, no 2, pp 762-770
- [12] IEEE Tutorial Course (1996) Optimal power flow: solution techniques, requirements and challenges. IEEE Power Engineering Society
- [13] Momoh JA, El-Haway ME, Adapa R (1999) A review of selected optimal power flow literature to 1993 Part 1 and Part 2. IEEE Transactions on Power Systems, vol 14 no 1, pp 96-111
- [14] Carpentier JL (1973) Differential injections method: A general method for secure and optimal load flows, Proceedings of IFAC Conference
- [15] Stott B, Marinho JL (1979) Linear programming for power system network security applications. IEEE Trans. on PAS, vol 98, no 3, pp 837-848
- [16] Alsac O, Bright J, Praise M, Stott B (1990) Further developments in LP-based optimal power flow. IEEE Transactions on Power Systems, vol 5, no 3, pp 697-711
- [17] Burchett RC, Happ HH, Wirgau KA(1982) Large scale optimal power flow. IEEE Trans. on PAS, vol 101, no 10, pp 3722-3732
- [18] Burchett RC, Happ HH, Veirath DR (1984) Quadratically convergent optimal power flow. IEEE Trans. on PAS, vol 103, no 11, pp 3267-3275
- [19] El-Kady MA, Bell BD, Carvalho VF, Burchett RC, Happ HH, Veirath DR (1986) Quadratically convergent optimal power flow. IEEE Trans. on Power Systems, vol. 1, no 2, pp 98-105
- [20] Glavitsch H, Sperry M (1983) Quadratic loss formula for reactive dispatch. IEEE Trans. on PAS, vol 102, no 12, pp 3850-3858
- [21] Sun DI, Ashley B, Brewer B, Hughes A, Tinney WF (1984) Optimal power flow by Newton approach. IEEE Trans. on PAS, vol 103, no 10, pp 2864-2880
- [22] Maria GA, Findlay JA (1987) A Newton optimal power flow program for Ontario Hydro EMS. IEEE Trans. on Power Systems, vol 2, no 3, pp 576-584

-
- [23] Tinny WF, Bright JM, Demaree KD, Hughes BA (1988) Some deficiencies in optimal power flow. *IEEE Trans. on Power Systems*, vol 3, no 2, pp 676-682
 - [24] Chang SK, Marks GE, Kato K (1990) Optimal real-time voltage control. *IEEE Trans. on Power Systems*, vol 5, no 3, pp 750-756
 - [25] Hollenstein W, Glavitch H (1990) Linear programming as a tool for treating constraints in a Newton OPF. *Proceedings of the 10th Power Systems Computation Conference (PSCC)*, Graz, Austria, August 19.-24.
 - [26] Karmarkar N (1984,) A new polynomial time algorithm for linear programming, *Combinatorica* 4, pp 373-395
 - [27] Vargas LS, Quintana VH, Vannelli A (1993) A tutorial description of an interior point method and its applications to security-constrained economic dispatch. *IEEE Trans. on Power Systems*, vol 8, no 3, pp 1315-1323
 - [28] Lu N, Unum MR (1993) Network constrained security control using an interior point algorithm. *IEEE Transactions on Power Systems*, vol 8, no 3, pp 1068-1076
 - [29] Zhang XP and Chen Z (1997) Security-constrained economic dispatch through interior point methods. *Automation of Electric Power Systems*, vol 21, no 6, pp 27-29
 - [30] Momoh JA, Guo SX, Ogbuobiri EC, Adapa R (1994) The quadratic interior point method solving power system optimization problems. *IEEE Trans. on Power Systems*, vol 9, no 3, pp1327-1336
 - [31] Granville S (1994) Optimal reactive power dispatch through interior point methods. *IEEE Transactions on Power Systems*, vol 9, no 1, pp 136-146
 - [32] Wu Y-C, Debs A, Marsten R E (1994) A direct nonlinear predictor-corrector primal-dual interior point algorithm for optimal power flows. *IEEE Trans. on Power Systems*, vol 9, no 2, pp 876-883
 - [33] Irisarri GD, Wang X, Tong J, Mokhtari S (1997) Maximum loadability of power systems using interior point nonlinear optimisation method. *IEEE Trans. on Power Systems*, vol 12, no 1, pp 167-172
 - [34] Wei H, Sasaki H, Yokoyama R (1998) An interior point nonlinear programming for optimal power flow problems within a novel data structure. *IEEE Trans. on Power Systems*, vol 13, no 3, pp 870-877
 - [35] Torres GL, Quintana VH (1998) An interior point method for non-linear optimal power flow using voltage rectangular coordinates. *IEEE Transactions on Power Systems*, vol 13, no 4, pp 1211-1218
 - [36] Zhang XP, Petoussis SG, Godfrey KR (2005) Novel nonlinear interior point optimal power flow (OPF) method based on current mismatch formulation. *IEE Proceedings—Generation, Transmission & Distribution*, to appear
 - [37] El-Bakry S, Tapia RA, Tsuchiya T, Zhang Y (1996) On the formulation and theory of the Newton interior-point method for nonlinear programming. *Journal of Optimisation Theory and Applications*, vol 89, no 3, pp 507-541
 - [38] La Scala M, Trovato M, Antonelli C (1998) On-line dynamic preventive control: An algorithm for transient security constraints. *IEEE Transactions on Power Systems*, vol 13, no 2, pp 601-610
 - [39] Gan D, Thomas RJ, Zimmermann RD (2000) Stability constrained optimal power flow. *IEEE Transactions on Power Systems*, vol 15, no 2, pp 535-540
 - [40] Chen L, Tada Y, et al (2001) Optimal operation solutions of power systems with transient stability constraints. *IEEE Transactions on Circuit and Systems – I: Fundamental Theory and Applications*, vol 48, no 3, pp 327-339

- [41] Yue Y, Kubokawa J, Sasaki H (2003) A solution of optimal power flow with multi-contingency transient stability constraints. *IEEE Transactions on Power Systems*, vol 18, no 3, pp 1094-1102
- [42] Rosehart W, Canizares C, Quintana VH (1999) Optimal power flow incorporating voltage collapse constraints. *Proceedings of the 1999 IEEE/PES Summer Meeting*, Edmonton, Alberta, July 1999, pp 820-825
- [43] Hingorani NG, Gyugyi L (2000) *Understanding FACTS – concepts and technology of flexible ac transmission systems*. New York: IEEE Press
- [44] Handschin E, Lehmkoester C (1999) Optimal power flow for deregulated systems with FACTS-Devices. *13th PSCC*, Trondheim, Norway, pp 1270-1276
- [45] Lehmkoester C (2002) Security constrained optimal power flow for an economical operation of FACTS-devices in liberalized energy markets. *IEEE Transactions on Power Delivery*, vol 17 no 2, pp 603-608
- [46] Acha E, H. Ambriz-Perez H (1999) FACTS devices modelling in optimal power flow using Newton's method. *13th PSCC*, Trondheim, Norway, pp 1277-1284
- [47] Zhang XP, Handschin E (2001) Advanced implementation of UPFC in a nonlinear interior point OPF. *IEEE Proceedings– Generation, Transmission & Distribution*, vol 148, no 3, pp 489-496
- [48] Zhang XP, Handschin E, (2001) Optimal power flow control by converter based FACTS controllers. *7th International Conference on AC-DC Power Transmission*, 28-30 Nov. 2001
- [49] Fardanesh B, Henderson M, Shperling B, Zelingher S, Gyugyi L, Schauder C, Lam B, Mounford J, Adapa R, Edris A (1998) Convertible static compensator: application to the New York transmission system. *CIGRE 14-103*, Paris, France, September
- [50] Fardanesh B, Shperling B, Uzunovic E, Zelingher S (2000) Multi-converter FACTS devices: the generalized unified power flow controller (GUPFC). *Proceedings of IEEE 2000 PES Summer Meeting*, Seattle, USA
- [51] Zhang XP, Handschin E, Yao MM (2001) Modeling of the generalized unified power flow controller in a nonlinear interior point OPF. *IEEE Trans. on Power Systems*, vol 16, no 3, pp 367-373
- [52] Zhang XP (2003) Modelling of the interline power flow controller and generalized unified power flow controller in Newton power flow. *IEEE Proc. - Generation, Transmission and Distribution*, vol 150, no 3, pp 268-274
- [53] Zhang XP (2004) Multiterminal voltage-sourced converter based HVDC models for power flow analysis. *IEEE Transactions on Power Systems*, vol 18, no 4, 2004, pp 1877-1884